The 15th International Conference on Gravitational Microlensing

Cosmic Equation of State from Strong Gravitational Lensing System

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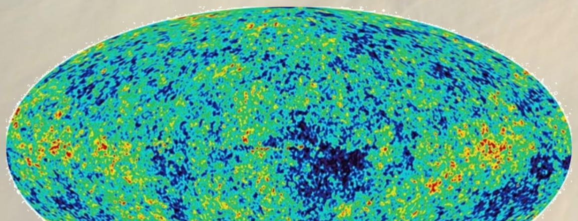
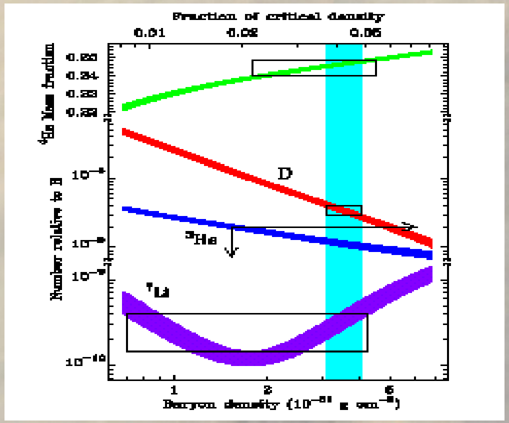
Pilars of Modern Cosmology

CMBR



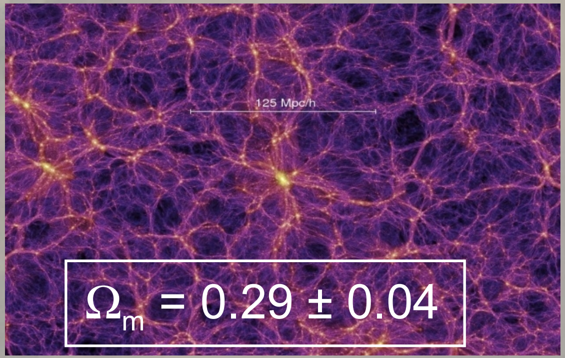
$$\Omega_{tot} = 1.003^{+0.013}_{-0.017}$$

BBN



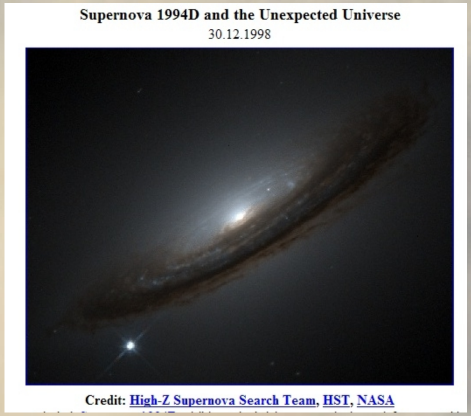
Ω_b = 0.042

LSS



Ω_m = 0.29 ± 0.04

SNIa on high redshifts



Gravitational Lensing

Introduction

- The explanation of the origin of dark energy is far from obvious and broadly speaking involves either invoking an unknown exotic component or modification of gravity at cosmological scales.

- Irrespective of theoretical approach chosen a common point with the observations usually occures at the level of w(z) coefficient in an effective equation of state for dark energy

$$p = w(z)\rho$$

- The power of modern cosmology lies in building up cosistency rather than in single experiment.

- Every alternative method of restricting cosmological parameters is desired

- We propse to use strongly gravitationally lensed systems in this context

this idea was discussed in *Biesiada M., 2006, Phys. Rev. D 73, 023006*

and in *Grillo et al., 2008, Astron. Astrophys., 477,397*

The Method

- our interest concentrate around: regime:strong & lens: galaxy

- the image separations in the system depend on angular diameter distances D_{ls} and D_{s}.

- angular diameter distances determined by background cosmology

$$D(z;p) = \frac{1}{1+z}\frac{c}{H_0}\int_0^z \frac{dz'}{h(z';p)}$$

dimensionless expansion rate

- spatial flatness is assumed (Hinshaw et al. 2009) $\Omega_{tot} = 1.0050^{+0.0060}_{-0.0061}$

- realistic lens model is needed ~ mass density profile approximeted by Singular Isothermal Sphere model (SIS)

The Method

- Einstein ring reads $$\theta_E = 4\pi \frac{\sigma_{SIS}^2}{c^2}\frac{D_{ls}}{D_s}$$
- σ_{SIS} lens velocity dispersion is well approximated by σ_o - central stellar velocity dispersion (see eg. Grillo et al. 2008)
- The main relation $$\frac{D_s}{D_{ls}} = \frac{4\pi\sigma_0^2}{c^2\theta_E} \leftarrow D^{obs}$$
- cosmological models enter through distance ratio

$$D^{th}(z_l, z_s, p) = \frac{D_s(p)}{D_{ls}(p)} = \frac{\int_0^{z_s}[dz'/h(z';p)]}{\int_{z_l}^{z_s}[dz'/h(z';p)]}$$

advantages of the method:

- independence on H_0
- not affected by dust absorption, or source evolutionary effect

- for observable counterpart we need realiable assesment of σ_o and θ_E
- cosmological parameters were fitted by minimizing $$\chi^2(p) = \sum_i \frac{(D_i^{obs} - D_i^{th}(p))^2}{\sigma_{D,i}^2}$$

Samples used

Lens ID	z_l	z_s	$\theta_E['']$	$\sigma_0\ [km/s]$
SDSS J0037-0942	0.1955	0.6322	1.47	282 ± 11
SDSS J0216-0813	0.3317	0.5235	1.15	349 ± 24
SDSS J0737+3216	0.3223	0.5812	1.03	326 ± 16
SDSS J0912+0029	0.1642	0.3240	1.61	325 ± 12
SDSS J0956+5100	0.2405	0.4700	1.32	318 ± 17
SDSS J0959+0410	0.1260	0.5349	1.00	229 ± 13
SDSS J1250+0523	0.2318	0.7950	1.15	274 ± 15
SDSS J1330-0148	0.0808	0.7115	0.85	195 ± 10
SDSS J1402+6321	0.2046	0.4814	1.39	290 ± 16
SDSS J1420+6019	0.0629	0.5352	1.04	206 ± 5
SDSS J1627-0053	0.2076	0.5241	1.21	295 ± 13
SDSS J1630+4520	0.2479	0.7933	1.81	279 ± 17
SDSS J2300+0022	0.2285	0.4635	1.25	305 ± 19
SDSS J2303+1422	0.1553	0.5170	1.64	271 ± 16
SDSS J2321-0939	0.0819	0.5324	1.57	245 ± 7
Q0047-2808	0.485	3.595	1.34	229 ± 15
CFRS03.1077	0.938	2.941	1.24	251 ± 19
HST 14176	0.810	3.399	1.41	224 ± 15
HST 15433	0.497	2.092	0.36	116 ± 10
MG 2016	1.004	3.263	1.56	328 ± 32

SLACS (SDSS rows) — LSD (Q0047-2808 to MG 2016)

$$\left\langle \frac{D_{ls}}{D_s} \right\rangle_{SLACS} = 0.58$$

- full sample n=20
- sub-sample n=7
- for comparison fit on Union08 sample – compilation of Kowalski et al. (2008) n=307 SNIa

Cosmological models tested

- ΛCDM

$w = -1$

$$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

$\mathbf{p} = \{\Omega_m\}$

- Quintessence

$w = const.$

$$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w)}}$$

Ω_m fixed $\mathbf{p} = \{w\}$

- Chevalier-Polarski-Linder

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

$$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_0+w_a)} \exp\left(\frac{-3 w_a z}{1+z}\right)}$$

Ω_m fixed $\mathbf{p} = \{w_0, w_a\}$

Results; fits on the full sample n=20

- Lens sample SLACS+LSD (n=15+5)

prior on Ω_m=0.27

Cosmological model	Best fit parameters (with 1σ)	χ^2/dof
ΛCDM	not possible	
Quintessence	$w = -0.9829 \pm 0.2415$	3.41
Chevalier-Linder-Polarski	$w_0 = 1.2605 \pm 0.8177$	3.05
	$w_a = -9.4443 \pm 4.4193$	

- Union08 SNIa sample (n=307)

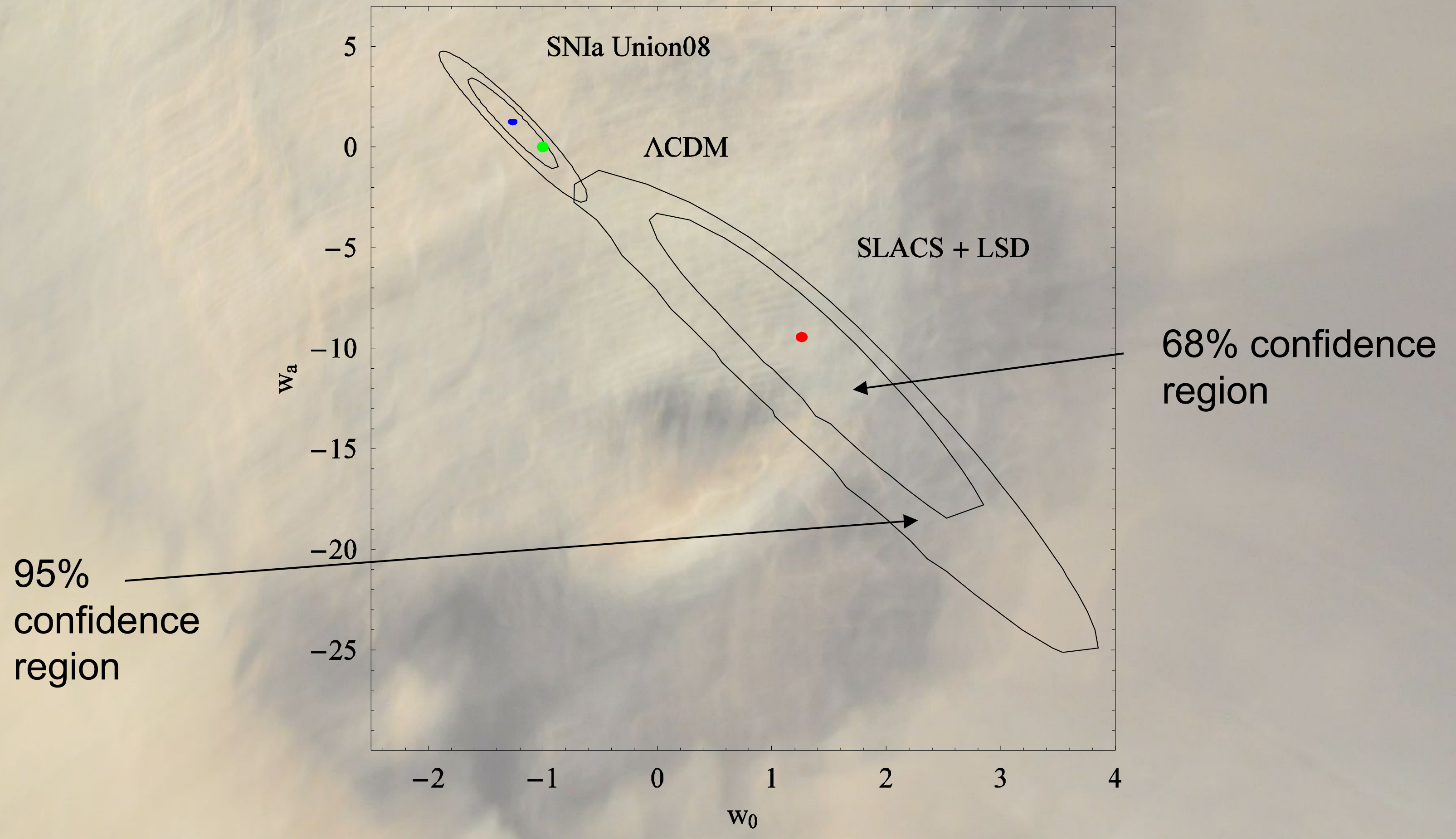
prior on Ω_m=0.27

Cosmological model	Best fit parameters (with 1σ)	χ^2/dof
ΛCDM	$\Omega_m = 0.287 \pm 0.027$	1.02
Quintessence	$w = -1.061 \pm 0.083$	1.02
Chevalier-Linder-Polarski	$w_0 = -1.263 \pm 0.257$	1.02
	$w_a = 1.254 \pm 1.484$	

•Quintessence : whole 2σ CI from SNIa in agreement with 1σ CI from lenses

$\langle -1.23, -0.85 \rangle$ $\langle -1.22, -0.74 \rangle$

Chevalier-Polarski-Linder: best fits and confidence regions



Results; fits on the restricted sample n=7

- on the restricted sample

  (n=7)

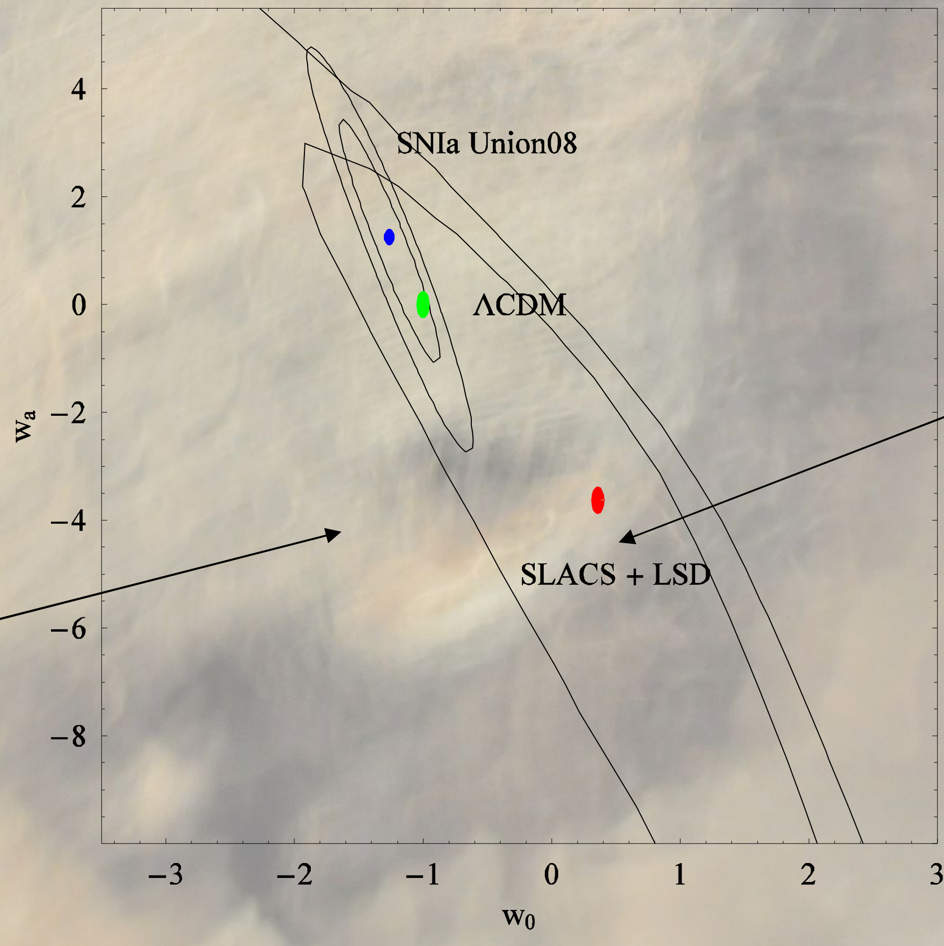
  prior on Ω_m=0.27

Cosmological model	Best fit parameters (with 1σ)	χ^2/dof
ΛCDM	$\Omega_m = 0.2660 \pm 0.2796$	1.76
Quintessence	$w = -0.6320 \pm 0.4461$	3.91
Chevalier-Linder-Polarski	$w_0 = 0.3588 \pm 1.2453$	1.88
	$w_a = -3.6301 \pm 5.3278$	

•ΛCDM fits – agreement with SNIa fits

•Quintessence: 2σ interval for the Union08 falls into 2σ interval for lenses

Chevalier-Polarski-Linder: best fits and confidence regions



68% confidence region

95% confidence region

standard rulers versus standard candles

- standard rulers
  - gravitational lenses (the same sample as before)

$$\chi^2(\mathbf{p}) = \sum_i \frac{[D_i^{obs} - D_i^{th}(\mathbf{p})]^2}{\sigma_{D,i}^2}$$

  - CMBR shift parameter R

$$R(\mathbf{p}) = \sqrt{\Omega_m} \int_0^{z_{lss}} \frac{dz}{h(z;\mathbf{p})} \qquad \chi^2_{CMB}(\mathbf{p}) = \frac{[R(\mathbf{p}) - 1.71]^2}{0.019^2}$$

  - BAO dimmensionless combination of so called dilatation scale

$$A(\mathbf{p}) = \frac{\sqrt{\Omega_m}}{0.35} \left[\frac{0.35}{h(0.35;\mathbf{p})} \left(\int_0^{0.35} \frac{dz}{h(z;\mathbf{p})} \right)^2 \right]^{1/3}$$

$$\chi^2_{BAO}(\mathbf{p}) = \frac{[A(\mathbf{p}) - 0.469]^2}{0.017^2}$$

- standard candles - SN Ia

$$\chi^2_{SN}(\mathbf{p}) = \sum_{i=1}^{N=307} \frac{[\mu^{obs}(z_i) - \mu^{th}(z_i;\mathbf{p})]^2}{\sigma_i^2}$$

μ distance modulus

joint analysis

The probes described above were combined by calculating joint likelihoods

$$L_{tot} = L_{rul} \times L_{cand} = L_{CMB} \times L_{BAO} \times L_{lens} \times L_{SN}$$

in our study equivalent to the assesment of

$$\chi^2_{tot}(\mathbf{p}) = \chi^2_{rul}(\mathbf{p}) \times \chi^2_{cand}(\mathbf{p}) = \chi^2_{CMB}(\mathbf{p}) \times \chi^2_{BAO}(\mathbf{p}) \times \chi^2_{lens}(\mathbf{p}) \times \chi^2_{SN}(\mathbf{p})$$

Standard rulers and standard candles probe distance measures based on different concepts – angular diameter distance and luminosity distance – so one step before making a full joint fit we performed fits based on rulers and candles separately

Two additional models tested (besides ΛCDM ($p=\Omega_m$), Quintessence ($p=\Omega_m, w$), CPL ($p=\Omega_m, w_0, w_a$))

- Chaplygin Gas $$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_{Ch}\left[A_0 + (1-A_0)(1+z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}}$$
- Braneworld scenario $$h(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_{r_C}} + \sqrt{\Omega_{r_C}}$$

Cosmological model	Best fit parameters	χ^2	Best fit parameters	χ^2
ΛCDM	$\Omega_m = 0.245 \pm 0.017$	$\chi^2 = 60.795$	$\Omega_m = 0.287 \pm 0.027$	$\chi^2 = 311.936$
Quintessence	$\Omega_m = 0.233 \pm 0.031$	$\chi^2 = 60.576$	$\Omega_m = 0.378 \pm 0.065$	$\chi^2 = 310.682$
	$w = -1.081 \pm 0.180$		$w = -1.360 \pm 0.329$	
Chevalier-Polarski-Linder	$\Omega_m = 0.255 \pm 0.049$	$\chi^2 = 60.198$	$\Omega_m = 0.270 \pm 0.652$	$\chi^2 = 310.914$
	$w_0 = -0.683 \pm 0.655$		$w_0 = -1.224 \pm 0.948$	
	$w_a = -1.252 \pm 2.0643$		$w_a = 1.511 \pm 4.849$	
Chaplygin Gas	$\Omega_m = 0.245 \pm 0.017$	$\chi^2 = 60.792$	$\Omega_m = 0.287 \pm 0.030$	$\chi^2 = 311.936$
	$A = 1.002 \pm 0.028$		$A = 0.999 \pm 0.042$	
	$\alpha = -1.704 \pm 1.357$		$\alpha = 0.001 \pm 0.097$	
Braneworld	$\Omega_m = 0.311 \pm 0.020$	$\chi^2 = 70.596$	$\Omega_m = 0.186 \pm 0.022$	$\chi^2 = 313.026$

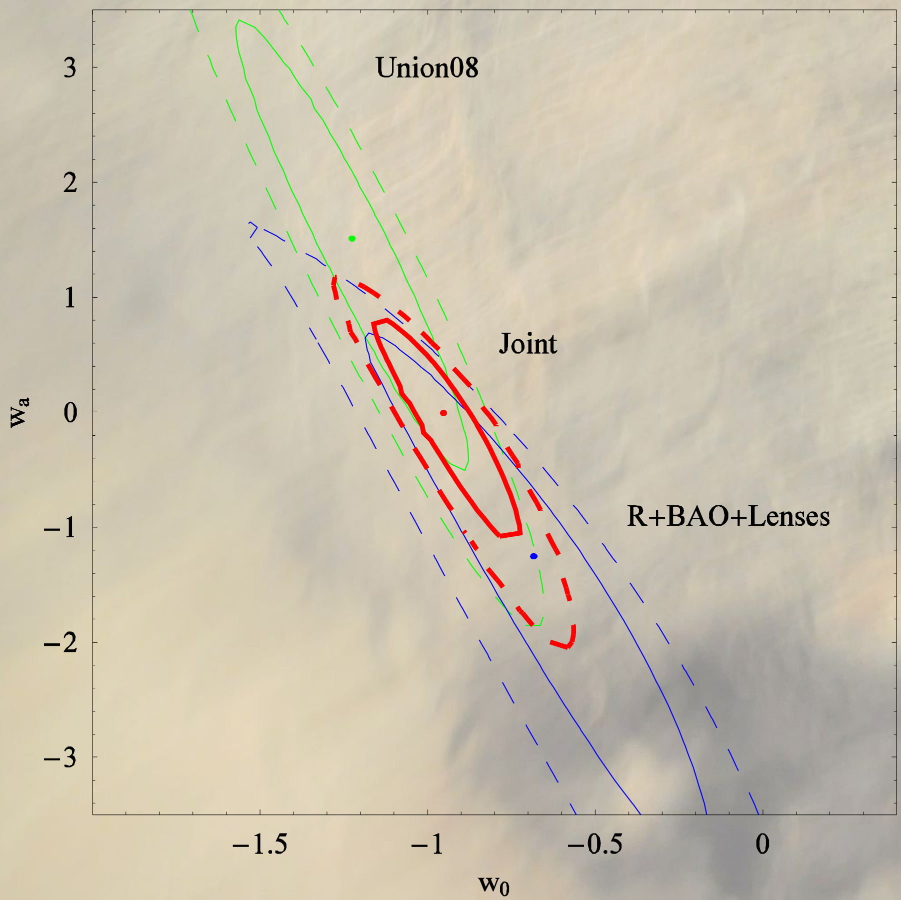
Cosmological model	Best fit parameters	χ^2
ΛCDM	$\Omega_m = 0.258 \pm 0.015$	$\chi^2 = 374.432$
Quintessence	$\Omega_m = 0.258 \pm 0.015$	$\chi^2 = 373.736$
	$w = -0.954 \pm 0.054$	
Chevalier-Polarski-Linder	$\Omega_m = 0.258 \pm 0.015$	$\chi^2 = 373.736$
	$w_0 = -0.953 \pm 0.145$	
	$w_a = -0.008 \pm 0.659$	
Chaplygin Gas	$\Omega_m = 0.258 \pm 0.015$	$\chi^2 = 373.732$
	$A = 0.950 \pm 0.088$	
	$\alpha = -1.102 \pm 1.815$	
Braneworld	$\Omega_m = 0.268 \pm 0.015$	$\chi^2 = 399.699$

Fits for:

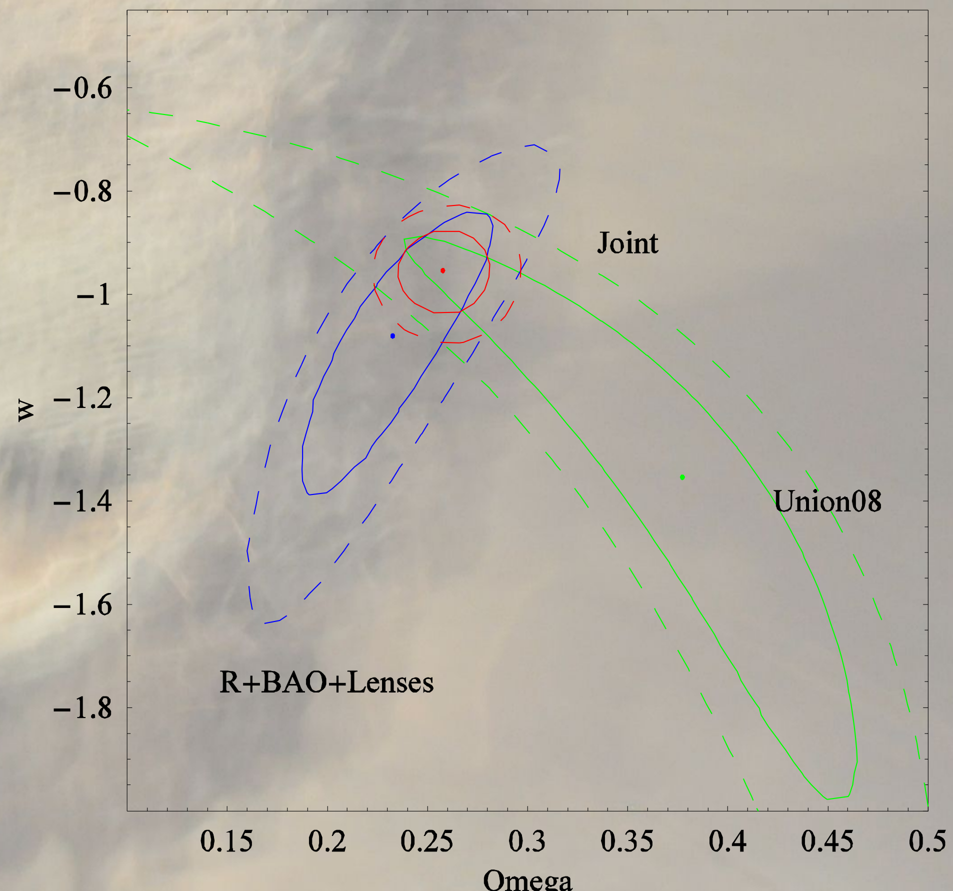
- rulers;
- candles
- joint

best fits (dots) and confidence regions

Chevalier-Polarski-Linder



Quintessence



Which model is the best?

- Minimizing the chi-square is good for finding best parameters in a model but is insufficient for deciding whether the model itself is the best one
- What we want to know is which model is supported by the data the best
- Here model selection is based on information theory
- We use two information-theoretic criteria:
  - Akaike Information Criterion (AIC),
  - Bayesian Information Criterion (BIC)

Akaike criterion is based on Kullback –Leibler information *I(f,g)* between two distributions

$$AIC = -2\log(L(\hat{p} \mid data)) + 2K$$

in our case: $$AIC = \chi^2(\hat{p} \mid data) + 2K$$

AIC value for a single model is meaningless, differences are used instead

$$\Delta_i = AIC_i - AIC_{\min}$$

Akaike weights – relative normalized likelihoods

Model	AIC	Δ_i	w_i	Odds against
ΛCDM	376.432	0.	0.497	1.
Quintessence	377.367	0.935	0.312	1.596
Chevalier-Polarski-Linder	379.736	3.304	0.095	5.217
Chaplygin	379.732	3.300	0.096	5.207
Braneworld	401.693	25.267	$1.62\ 10^{-6}$	$3.07\ 10^{5}$

Likelihood function

$$\mathcal{L}(\hat{p} \mid data) \propto \exp[-\frac{1}{2}\Delta_i]$$

Bayesian Information Criterion (BIC)

$$BIC = -2\ln(\mathcal{L}(\hat{p} \mid data)) + 2K\ln(n)$$

number of parameters

sample size

Model	BIC	$BIC\Delta_i$	BIC w_i	BIC Odds against
ΛCDM	380.228	0.	0.907	1.
Quintessence	384.959	4.731	0.085	10.650
Chevalier-Polarski-Linder	391.124	10.896	0.004	232.307
Chaplygin	391.120	10.892	0.004	231.842
Braneworld	405.495	25.267	$2.96\ 10^{-6}$	$3.07\ 10^{5}$

derived by Schwartz

Conclusions

- Obtained results demonstrate possibility of practical use of strong gravitational system for constraining cosmological models
- The small number of lenses available (at the time we started our study - 2009) makes the precision of cosmological parameters determination poor comparing with oher methods, yet feasibility of the method is demonstrated.
- Over last year the SLACS sample of lenses with realiable data on σ_o and θ_E has grown up to 58 .
- Grillo et al. 2008 demonstrated on simulations that a sample of 100 or 200 lensing systems would be enough to give competitive constraints (constraints on Ω_Λ) .
- Work on actually available sample is in progress.
- Presented results are also available in the paper:

  Biesiada M., Piórkowska A., Malec B., ***MNRAS***, 406,1055-1059 (2010)

Conclusions

- The best fit obtained for the model parameters in joint analysis is in agreement with joint analyses performed by others on different set of diagnostic probes.
- Information theoretic methods used to assess which model is the most supported by data lead to conclusion that the concordance model ΛCDM is preferred and brane world scenario is practically irrelevant.
- AIC :
  - ΛCDM is only slighty prefered over quintessence
  - CPL an Chaplygin are considerably less supported
  - Braneworld ruled out
- BIC:
  - ΛCDM wins
  - Quintessence considerably less supported
  - Others – ruled out

Biesiada M., Malec B., Piórkowska A. ***RAA*** submitted (2011)