

The 15<sup>th</sup> Conference on Gravitational Microlensing

Salerno, 22.1.2010

## **Light Curve Errors**

### **Introduced by Limb-darkening Models**

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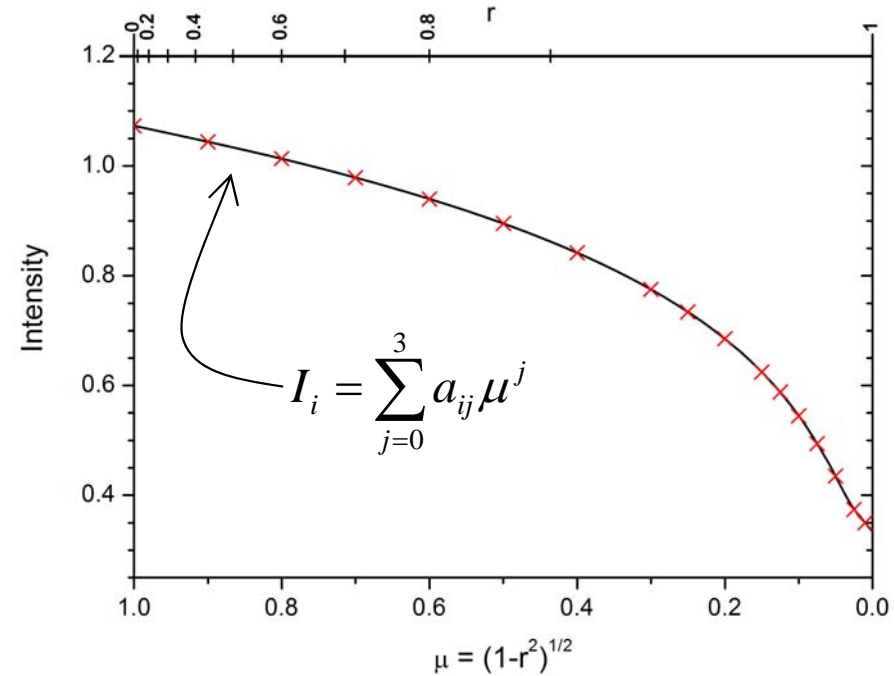
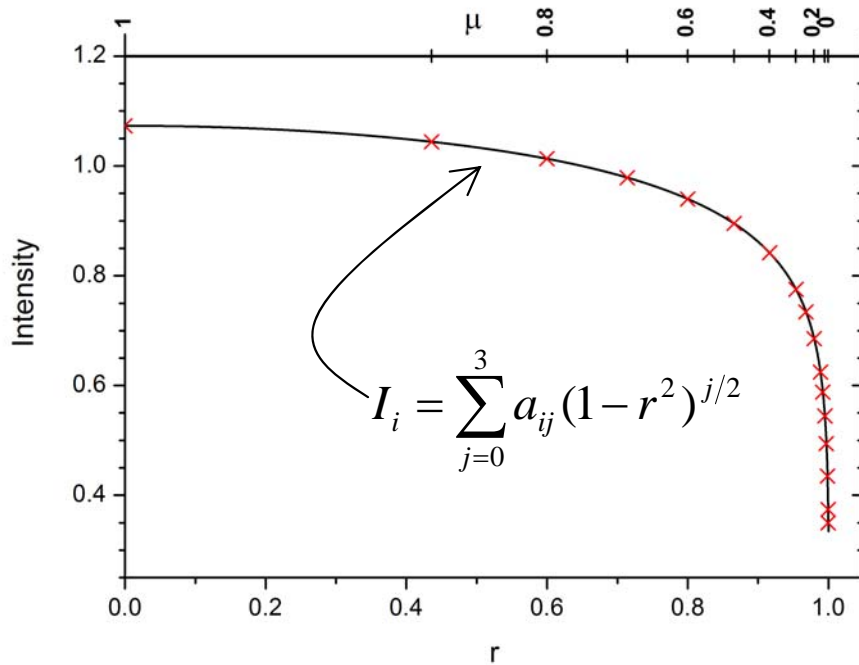
Charles University, Prague

# Outline

1. Microlensing a model atmosphere limb-darkening profile
2. Microlensing limb-darkening model fits to profile
3. Amplification residuals
4. Statistical results for Kurucz's ATLAS9 grid in *BVRI*
5. Residual patterns for observed events
6. Sensitivity of binary vs. single-lens caustic-crossing events

# Kurucz model intensity profiles

ATLAS9 grid - 9581 stellar model atmospheres ( $T_{\text{eff}}=3500..50000$  K);  
 2 nm resolution in optical; intensity for 17 rays



To compute microlensed flux evaluate

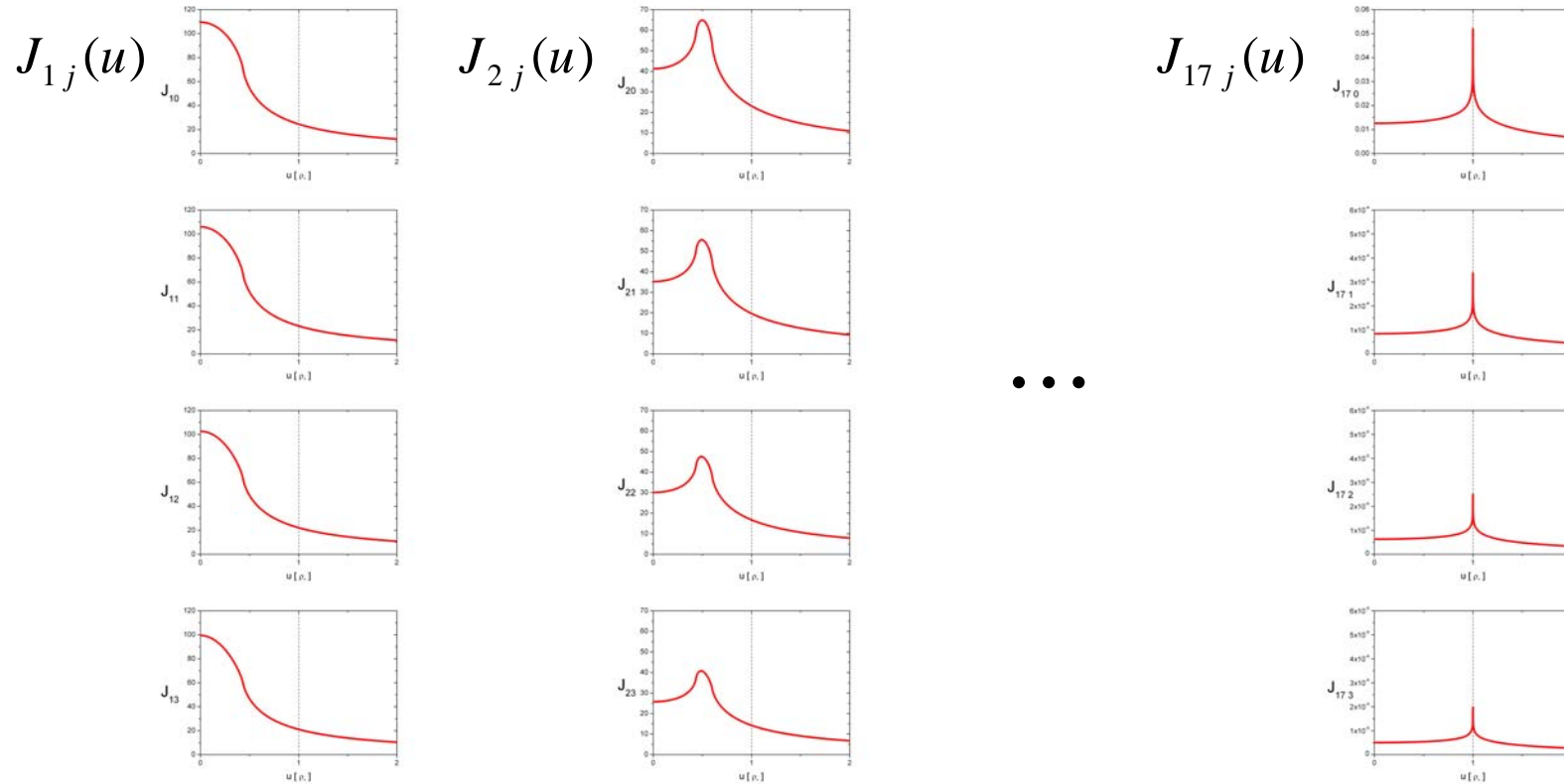
$$J_{ij}(u) = \int_{r_i}^{r_{i+1}} (1-r^2)^{j/2} A(u, r) r dr$$

$i = 1..17, j = 0..3$

angle-integrated point-source amplification  
 (analytical expression e.g. Heyrovský 2003)

# Computation of Kurucz profile amplification

numerically + analytically across  $r=u/\rho_*$  ( for  $0 < u/\rho_* < 2$ ,  $\rho_* = 0.025$  )



Amplification

$$A_{KURUCZ}(u) = \frac{\sum_{ij} a_{ij} J_{ij}(u)}{2\pi \sum_{ij} a_{ij} \frac{\mu_i^{j+2} - \mu_{i+1}^{j+2}}{j+2}}$$

# Amplification residuals for LD models

Linear  $I(r) = I_0[1 - (1 - \nu\sqrt{1-r^2})]$

PCA-2  $I(r) = I_0[f_1(r) + \nu f_2(r)]$

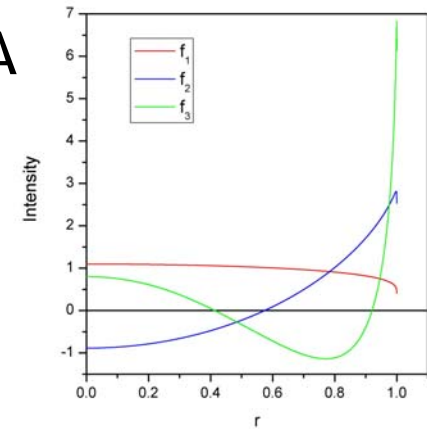
Quadratic  $I(r) = I_0[1 - \nu_1(1 - \sqrt{1-r^2}) - \nu_2(\sqrt{1-r^2})^2]$

Square-root  $I(r) = I_0[1 - \nu_1(1 - \sqrt{1-r^2}) - \nu_2(1 - \sqrt[4]{1-r^2})]$

Logarithmic  $I(r) = I_0[1 - \nu_1(1 - \sqrt{1-r^2}) - \nu_2\sqrt{1-r^2} \ln\sqrt{1-r^2}]$

PCA-3  $I(r) = I_0[f_1(r) + \nu_1 f_2(r) + \nu_2 f_3(r)]$

PCA



Need to evaluate only 2 more

$$J_{3/2}(u) = \int_0^1 (1-r^2)^{1/4} A(u,r) r dr$$

$$J_{2L}(u) = -\int_0^1 (1-r^2)^{1/2} \ln\sqrt{1-r^2} A(u,r) r dr$$

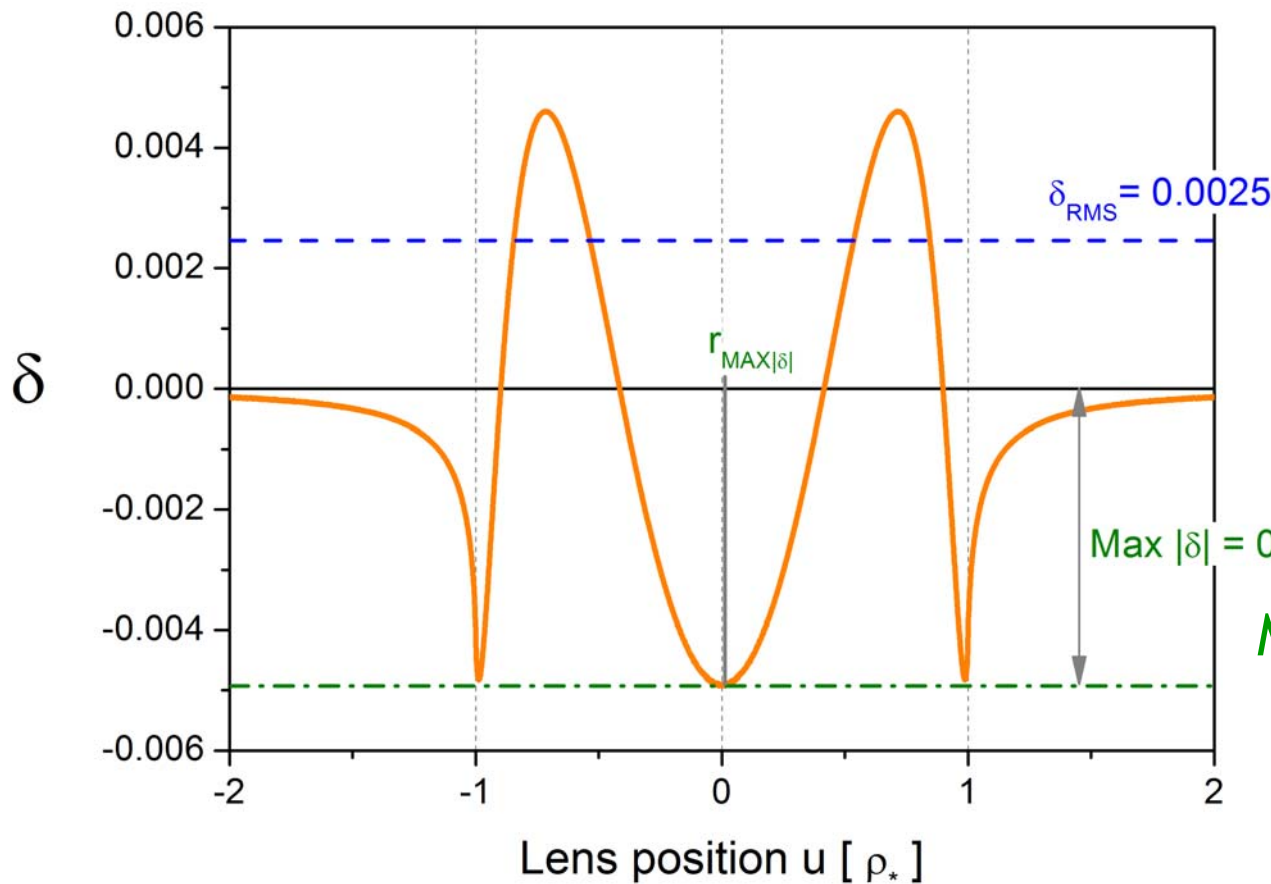
Residual

$$\delta(u) = \frac{A_{MODEL}(u)}{A_{KURUCZ}(u)} - 1$$

# Amplification residuals – quantities to check

Example:  $T_{\text{eff}}=3500\text{K}$ ,  $\log g=0$ ,  $[\text{Fe}/\text{H}]=1$ ,  $v_t=2 \text{ km/s}$ , V band

LD model: linear



r.m.s. residual

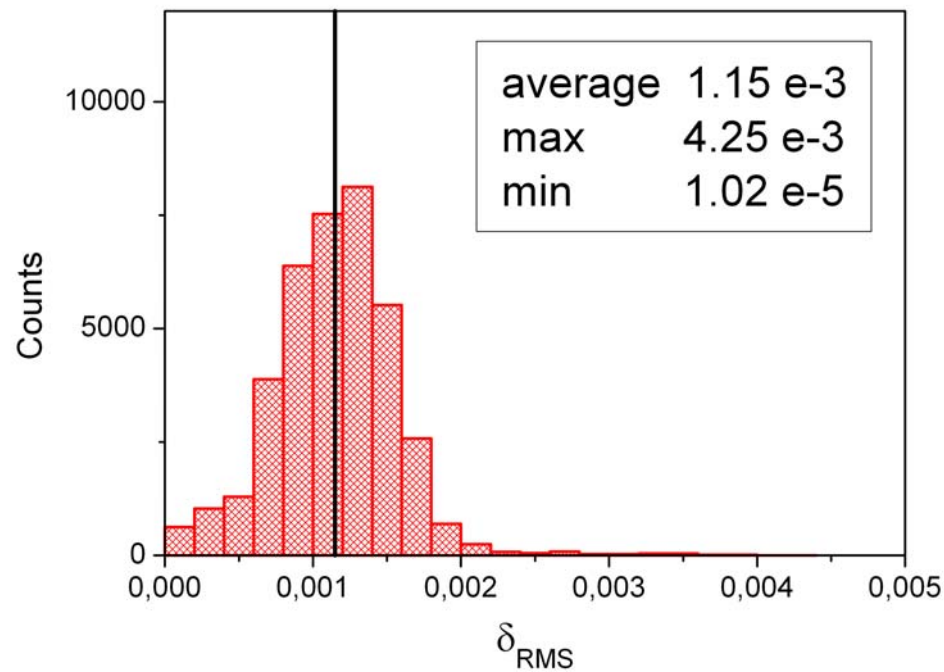
$$\delta_{RMS} = \sqrt{\frac{1}{2} \int_0^2 \delta^2 \left(\frac{u}{\rho_*}\right) d\frac{u}{\rho_*}}$$

Max. abs. residual  
+ position

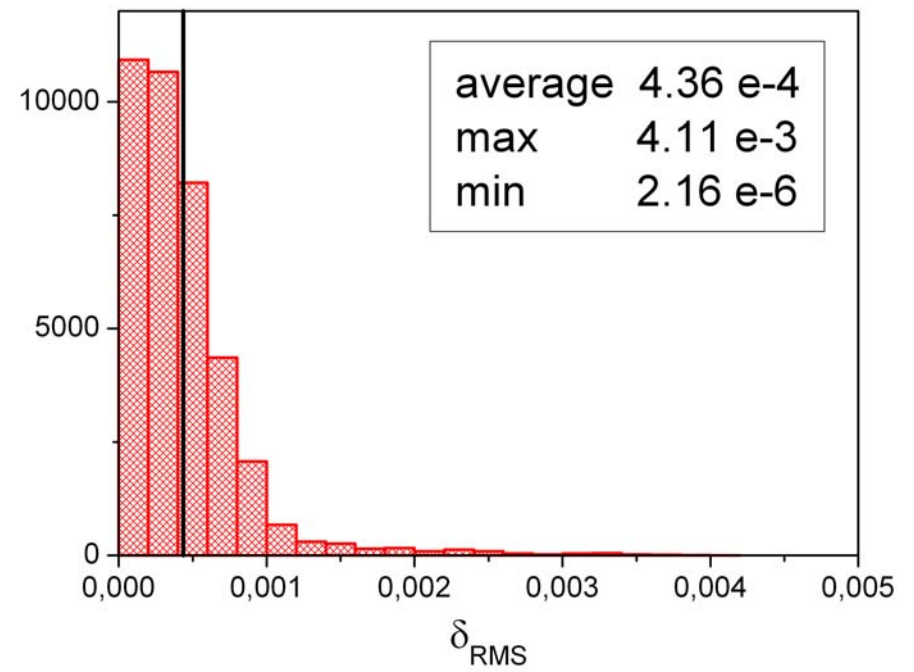
$$\begin{matrix} \text{Max } |\delta| \\ r_{\text{Max}|\delta|} \end{matrix}$$

# Full Kurucz grid in *BVR*: 1-par. LD r.m.s. residual

## Linear LD



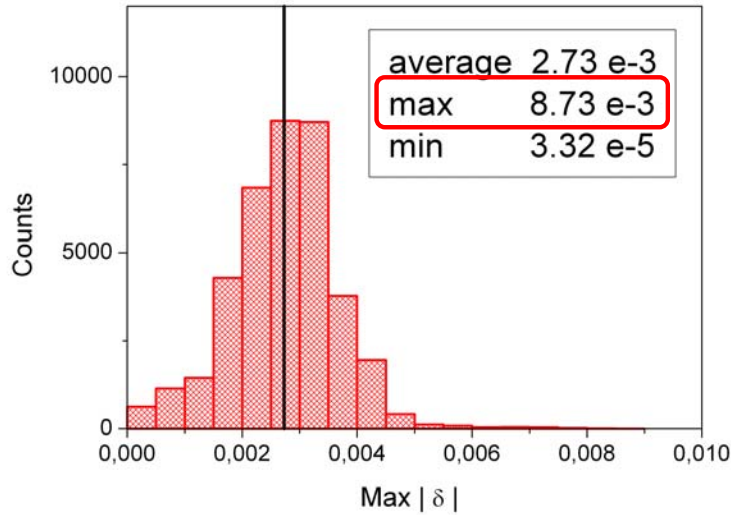
## PCA-2 LD



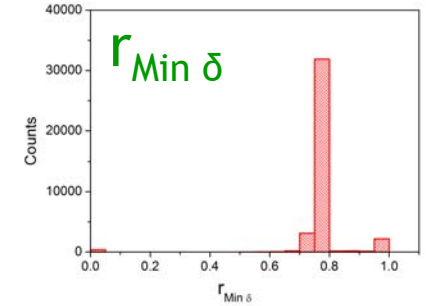
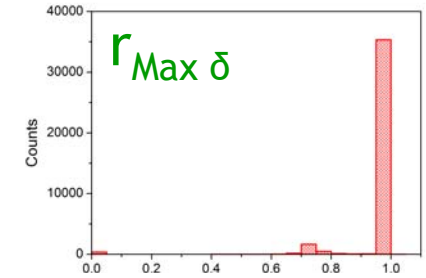
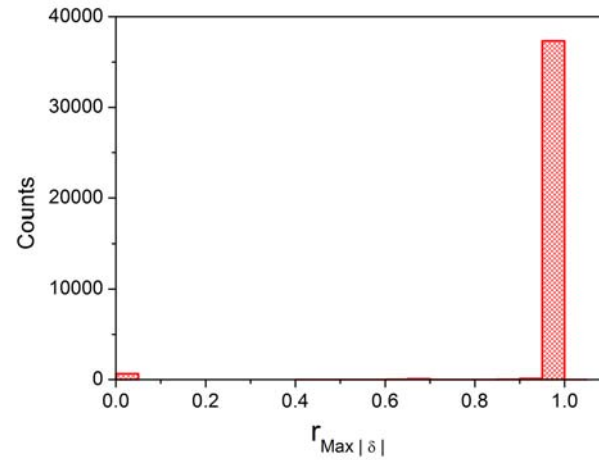
# Full Kurucz grid in *BVRi*: 1-par. LD max. residual

Linear LD

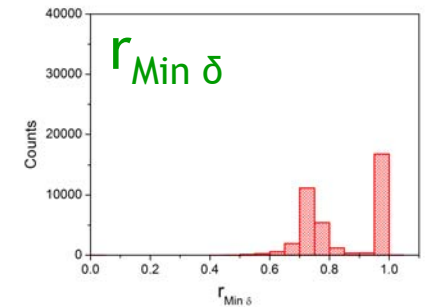
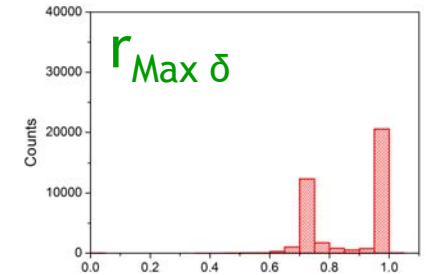
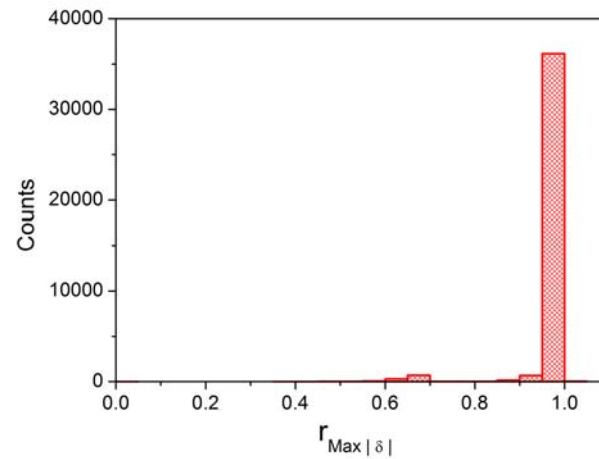
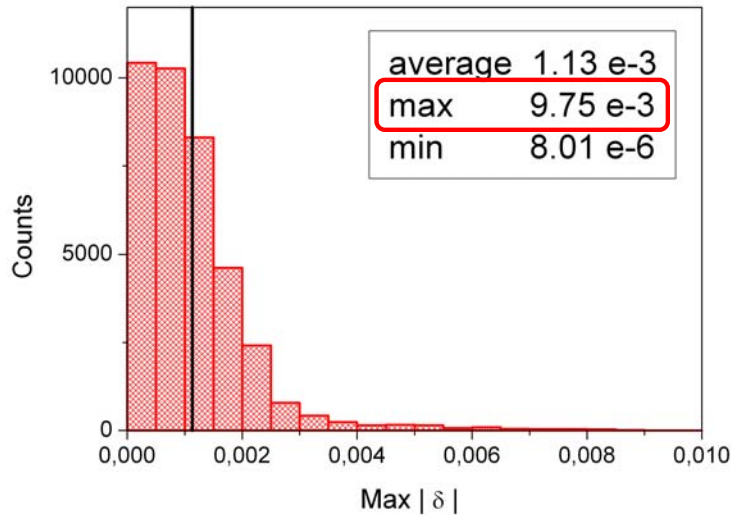
Max.  $|\delta|$



$r_{\text{Max}|\delta|}$



PCA-2 LD





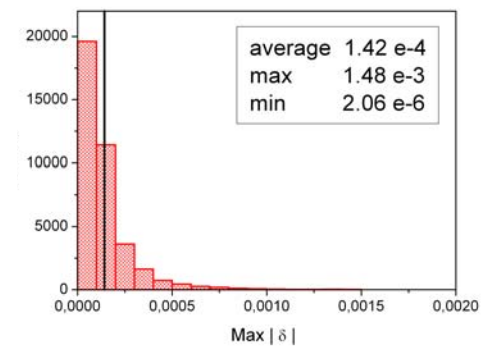
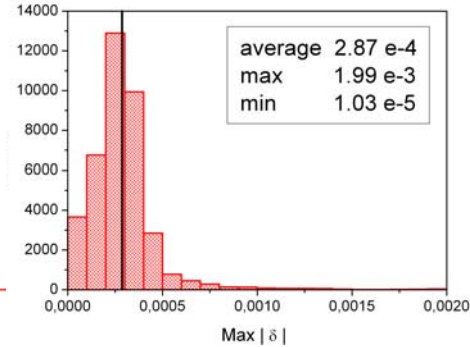
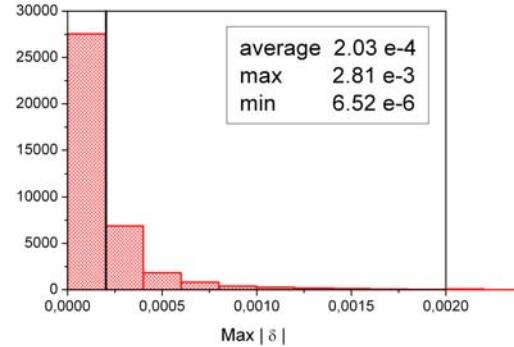
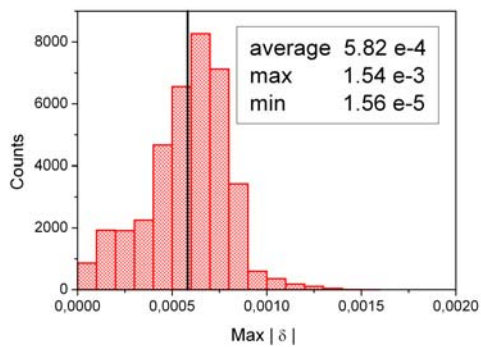
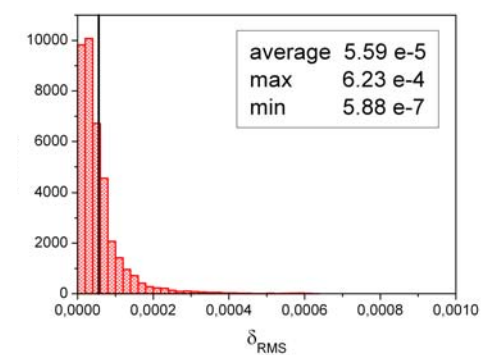
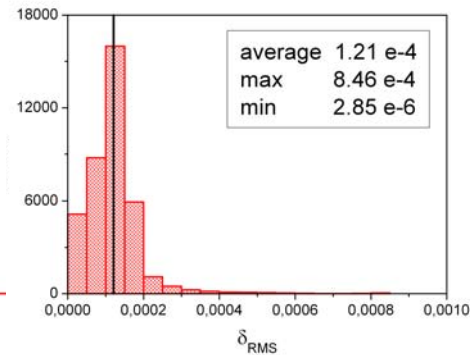
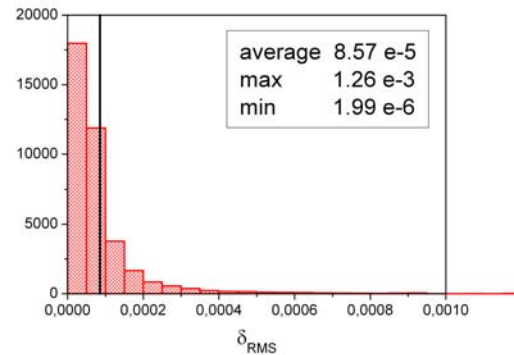
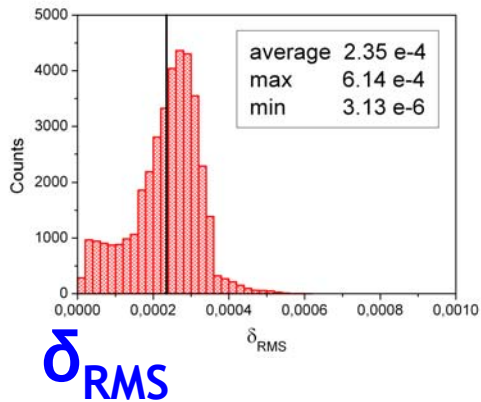
# Full Kurucz grid in *BVRi*: 2-par. LD models

Quadratic

Square-root

Logarithmic

PCA-3



Max.  $|\delta|$

ave  $5.82\text{e-}4$

max  $1.54\text{e-}3$

$2.03\text{e-}4$

$2.81\text{e-}3$

$2.87\text{e-}4$

$1.99\text{e-}3$

$1.42\text{e-}4$

$1.48\text{e-}3$

# Observed microlensing events analyzed for LD

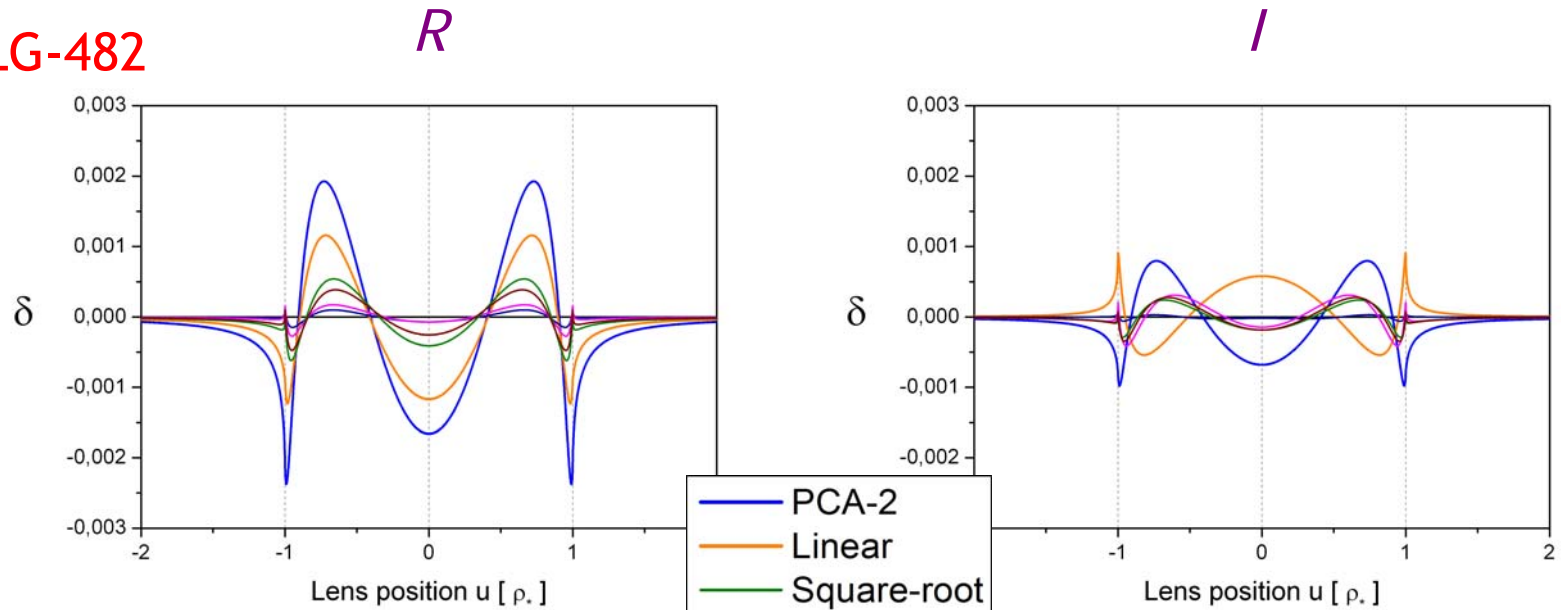
## OGLE 2004-BLG-482

(Zub et al. 2011)

$T_{\text{eff}} = 3750 \text{ K}$

$\log g = 2.0$

$[\text{Fe}/\text{H}] = 0$



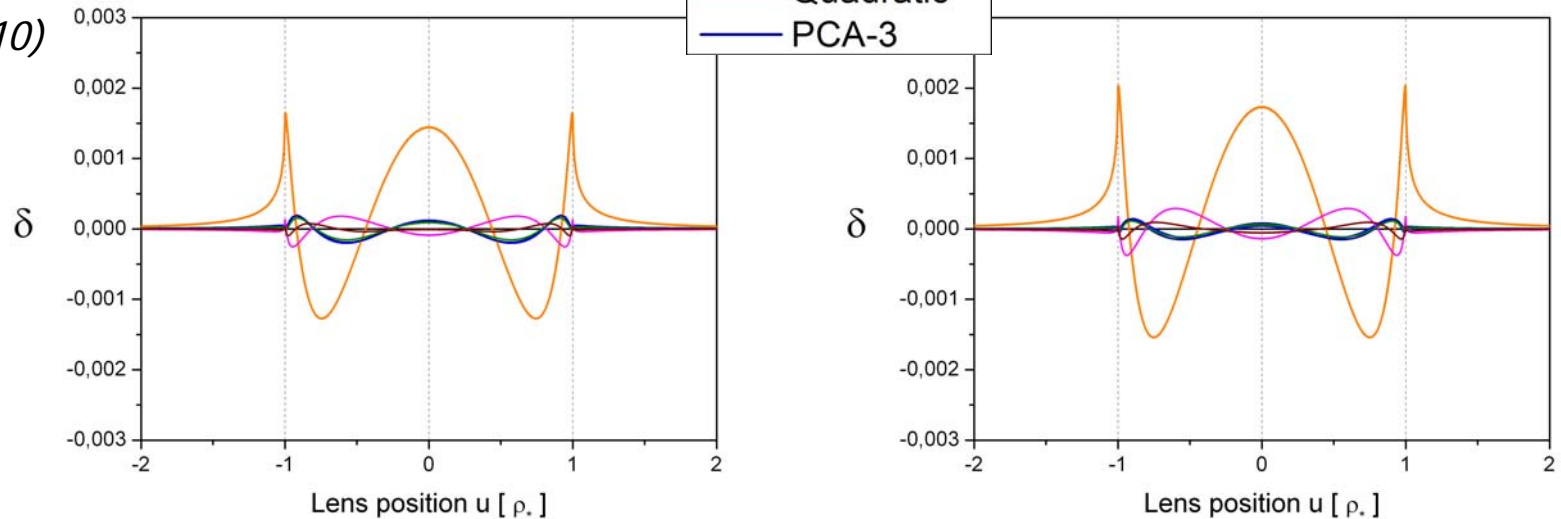
## OGLE 2008-BLG-290

(Fouqué et al. 2010)

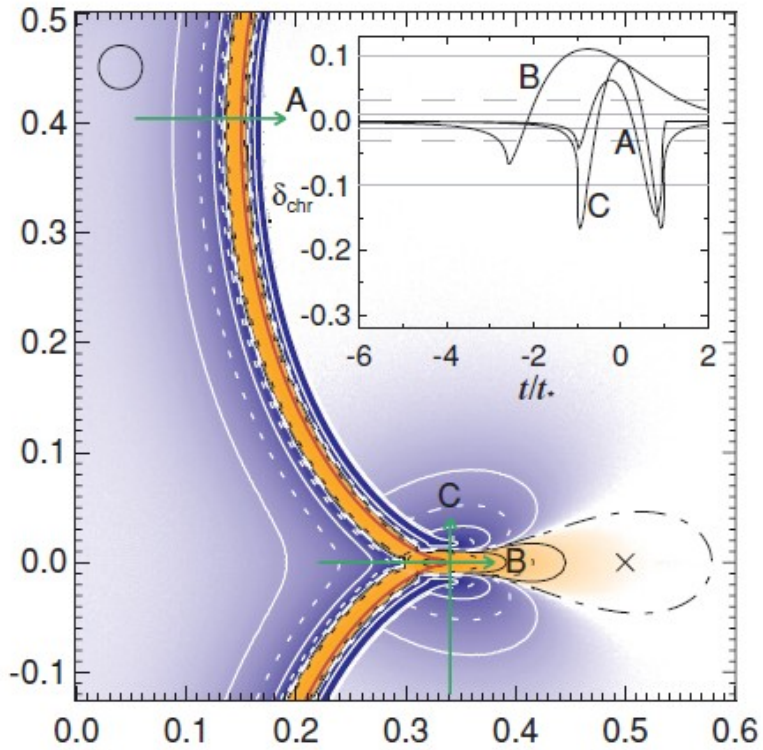
$T_{\text{eff}} = 4750 \text{ K}$

$\log g = 3.0$

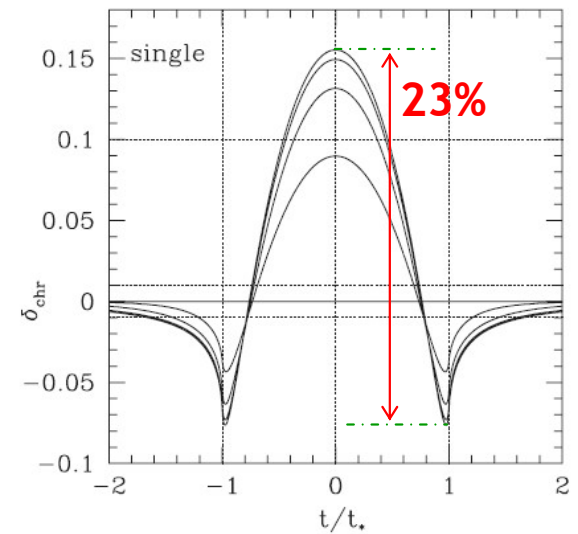
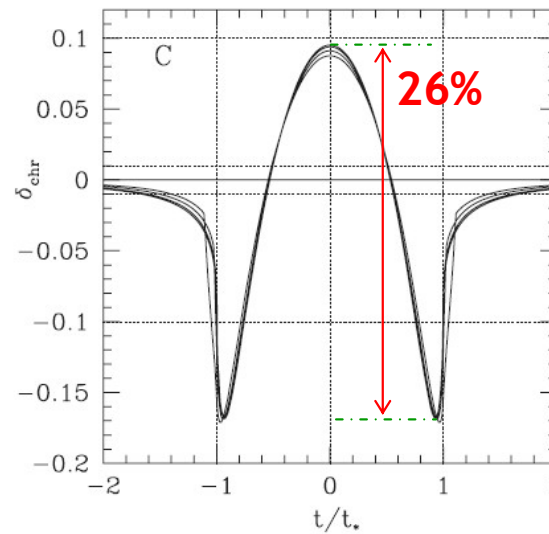
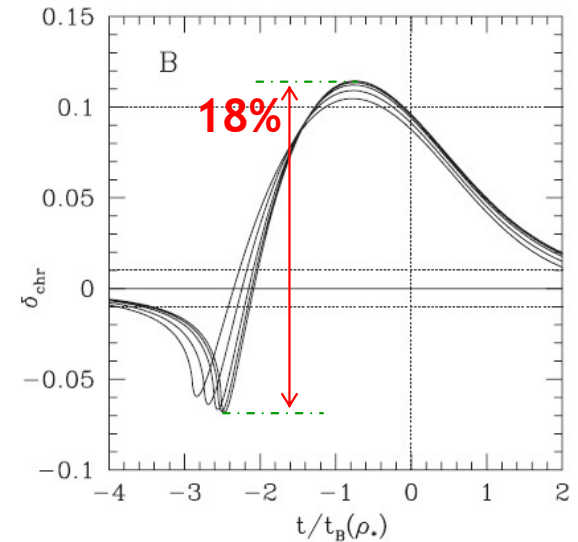
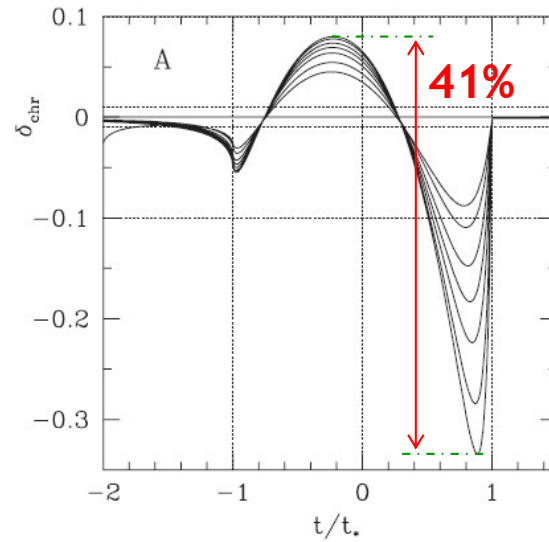
$[\text{Fe}/\text{H}] = 0.3$



# Binary lenses: comparison of sensitivity



(Pejcha & Heyrovský 2009)



⇒ sensitivity comparable

# Conclusions

- check r.m.s. residual for relevant stellar atmosphere parameters
- estimate effect on  $\chi^2$  from photometric accuracy and # of points
- if necessary, use higher order LD model
- alternatively, use stellar model atmosphere LD profile in analysis

(as in MOA 2009-BLG-266)