Critical Curve Topology in Special Triple Lens Configurations

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Abstract: Inspired by the Erdl & Schneider [1] analysis of the parameter dependence of binary lensing topologies, we extend their approach to special cases of the triple lens. While the binary lens is characterised by two parameters, three more parameters are needed to describe the triple lens. We analysed several two-dimensional cuts through the five-dimensional parameter space, identifying the boundaries of regions with different critical curve topology. For each region we present corresponding critical curves and caustics.

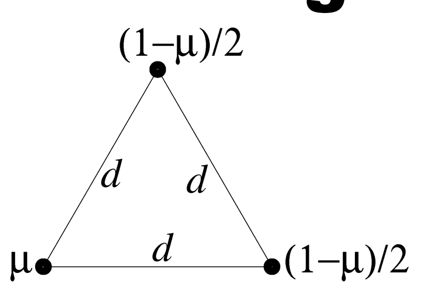
Theory: The triple lens equation in complex notation with relative masses μ_i at positions z_i is given by (1). Topology changes (mergers/splits) occur when saddle points (2) of the Jacobian lie on the critical curve (3). Corresponding parameter combinations (lines in plots at lower left) can be obtained by the Sylvester matrix method for finding conditions for a common root of (2) and (3).

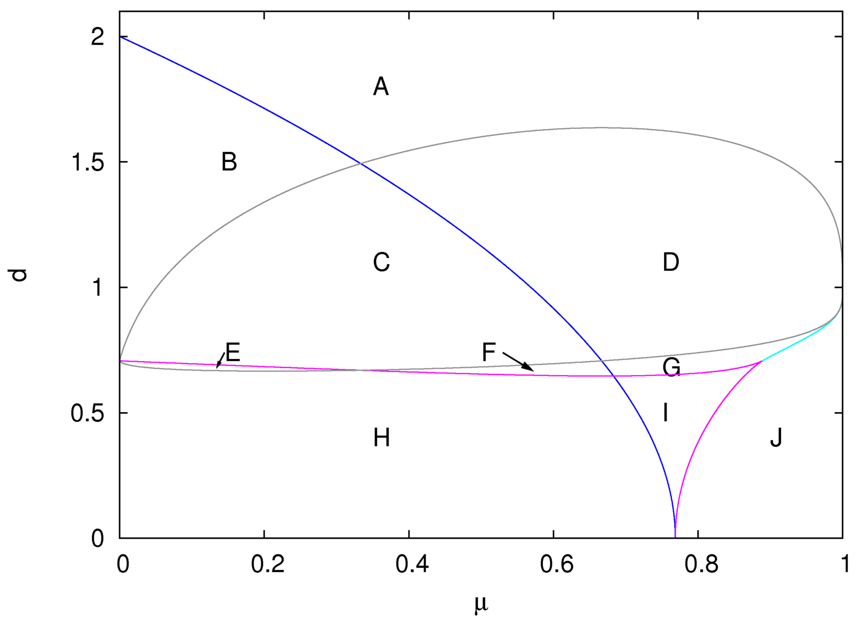
$$\zeta = z - \frac{\mu_1}{(\bar{z}-\bar{z}_1)} - \frac{\mu_2}{(\bar{z}-\bar{z}_2)} - \frac{\mu_3}{(\bar{z}-\bar{z}_3)} \qquad (1)$$

$$\frac{\mu_1}{(z-z_1)^3} + \frac{\mu_2}{(z-z_2)^3} + \frac{\mu_3}{(z-z_3)^3} = 0 \qquad (2)$$

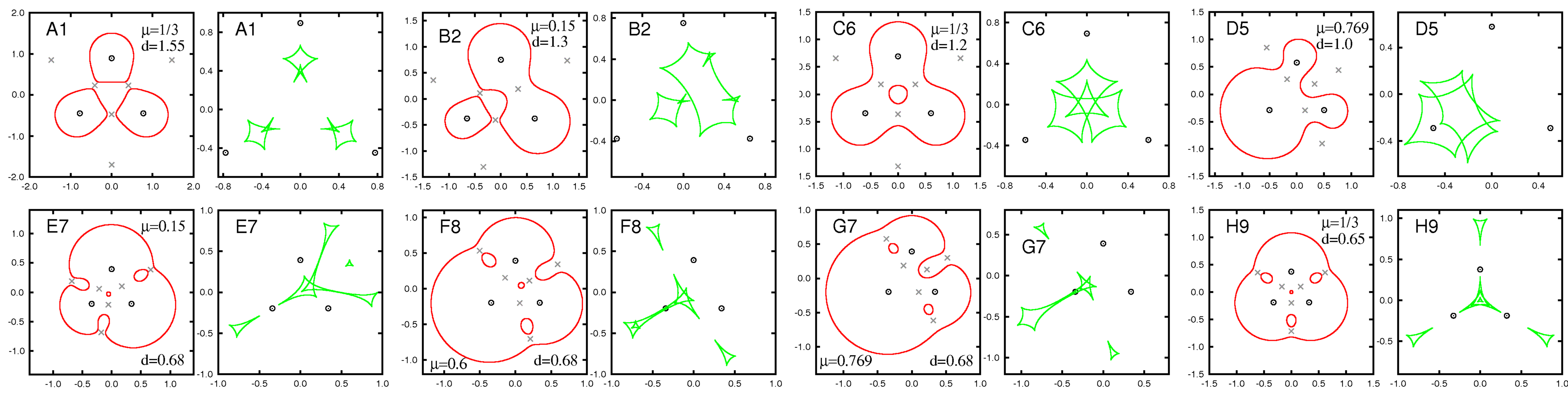
$$\frac{\mu_1}{(z-z_1)^2} + \frac{\mu_2}{(z-z_2)^2} + \frac{\mu_3}{(z-z_3)^2} = e^{-2i\phi} \qquad (3)$$

Equilateral triangle (μ,d)

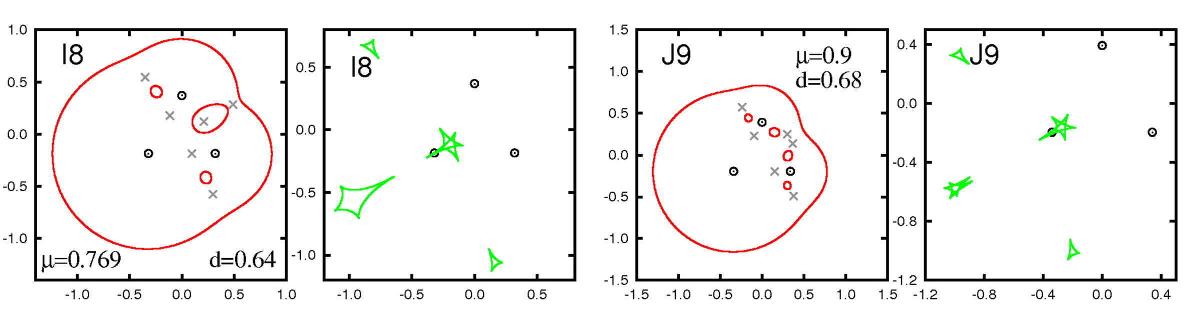




Parameter Space: Ten regions marked by letters. Corresponding critical curves and caustics in panels at right. Polynomial curves: blue and magenta - 6th degree in d^2, cyan - 3rd in d^4, grey - 12th in d^4.

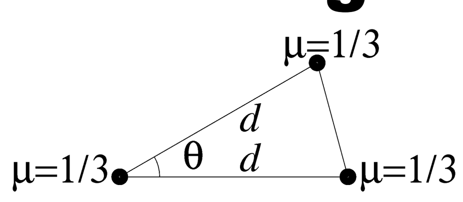


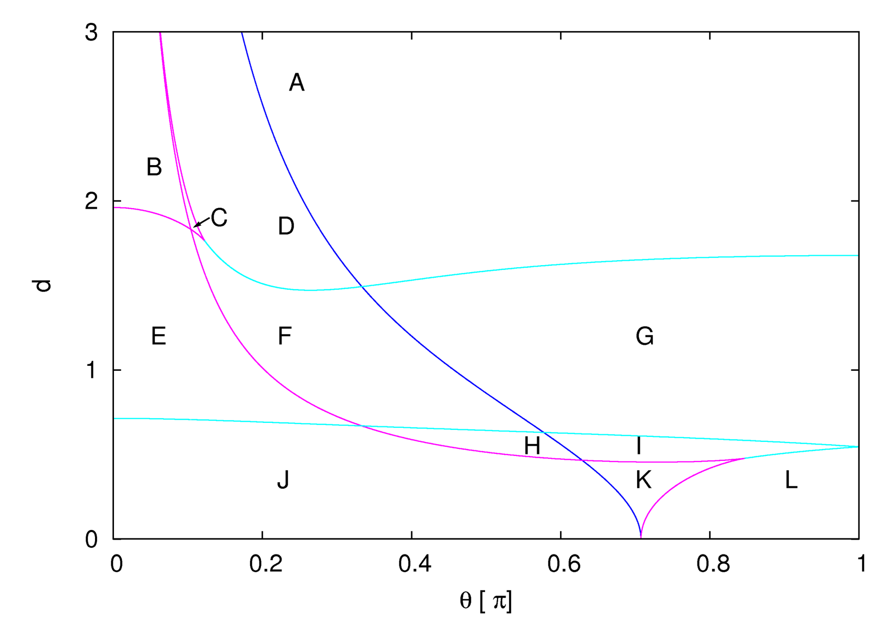
Critical curves and **caustics** : *Black circles* - lens positions; *grey crosses* - Jacobian saddle points; *letter in upper left corner* - region of parameter space (see left plot); *number in upper left corner* - critical curve topology type. Lens parameters marked next to critical curve. Caustics are obtained from (1). Seven different topologies in this case.



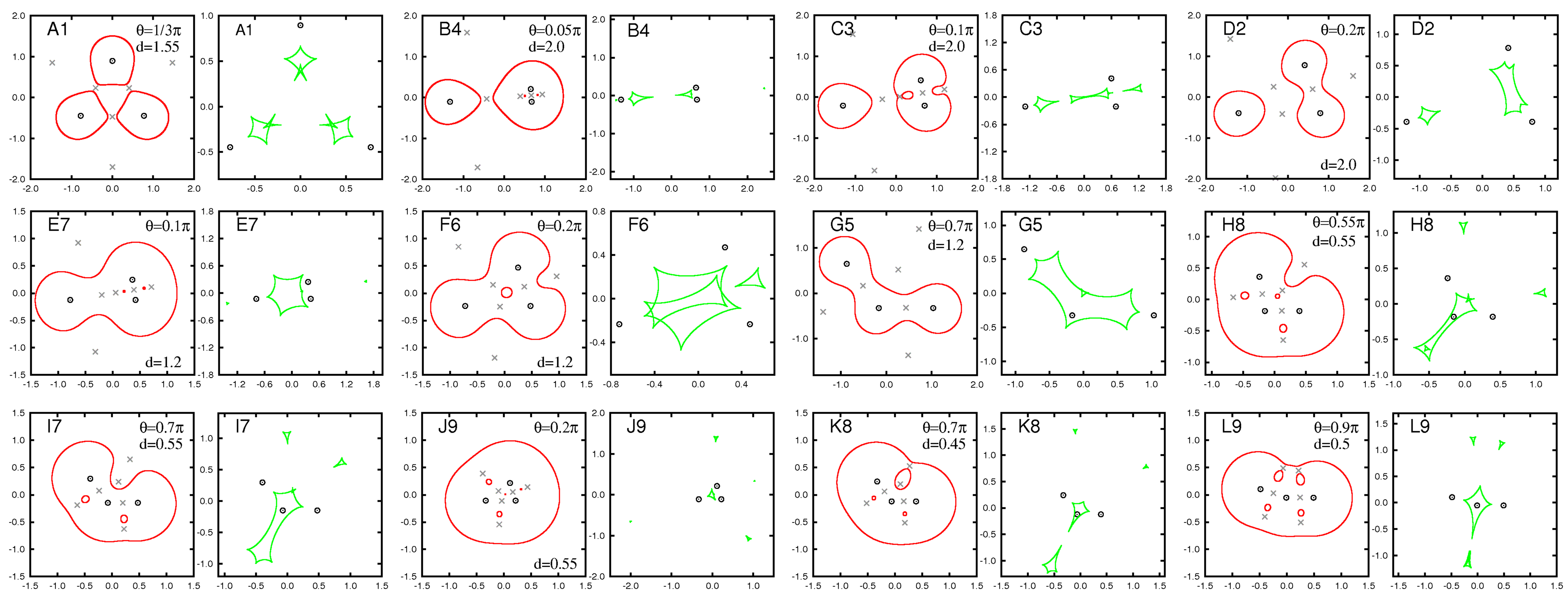
Limiting Cases: Point lens μ=1; d=0. Binary lens μ=0. Three independent lenses d→∞.

Isosceles triangle (θ,d)





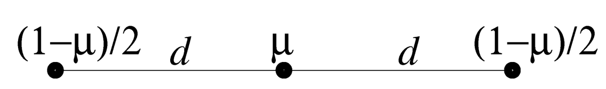
Parameter Space: Twelve regions. Polynomial curves: blue and magenta - 6th degree in d^2, cyan - 15th in d^4.

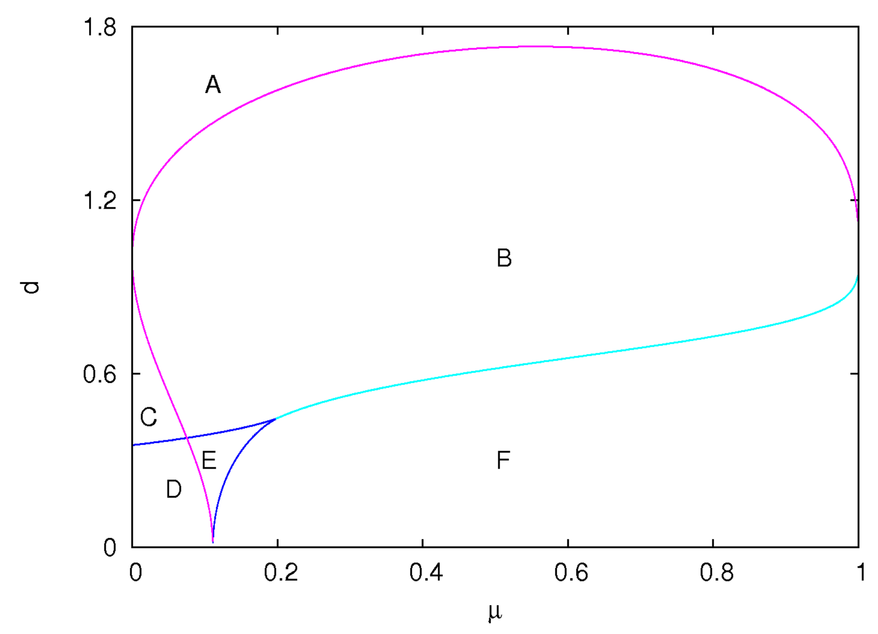


Critical curves and **caustics** (notation as above): Nine different topologies

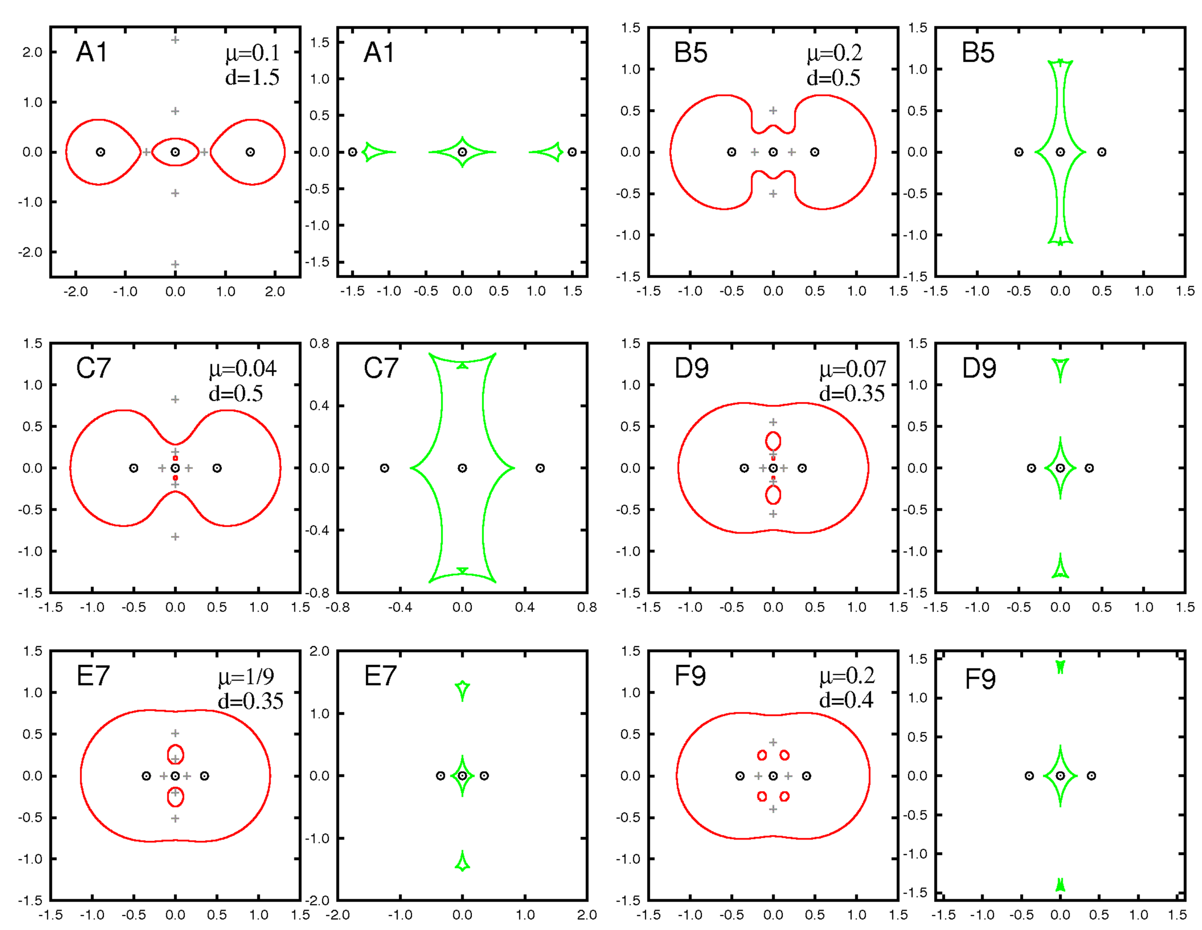
Limiting Cases: Point lens d=0. Binary lens θ=0. Point lens+binary lens d→∞ , $\theta \sim d^{-1}$. Three independent lenses θd→∞.

Linear (μ,d)





Parameter Space: Six regions. Polynomial curves: blue and magenta - 3rd degree in d^2, cyan - 3rd in d^4.



Critical curves and **caustics** (notation as above): Four different topologies

Limiting Cases: Point lens d=0; μ=1. Binary lens μ=0. Three independent lenses d→∞.

Acknowledgements

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References

[1] Erdl, H., Schneider, P., 1993, A&A, 268, 453

[2] Danek, K., 2010, Charles University in Prague (Master's Thesis)