Selected topics on the functional renormalization group and its applications

Istituto Nazionale di Fisica Nucleare

Gian Paolo Vacca INFN - Bologna

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Outline

- Functional RG approach to QFTs
	- Perturbative
	- Wilsonian (non perturbative)
- Multicritical Yukawa theories
- Applications to Hamiltonian systems:
	- Quantum mechanics
	- RFT for Regge limit of QCD
- Conclusions

Introduction

Physical systems, very different at microscopic level, can show phases characterized by the same Universal behavior when the correlation length diverges (2nd order phase transition).

Example: Ising model Critical phenomena are conveniently described by Quantum and Statistical Field Theories.

Most famous example:

3D Ising universality class (Magnetic systems, Water) in a Landau-Ginzburg description as a scalar QFT,

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RG is the proper tool to investigate related questions

Theory space (fields and symmetries)

The critical theories are points in a suitable theory space characterized by scale invariance. If there is Poincare' invariance it is often lifted to conformal invariance

RG

In a Renormalization Group description critical field theories are associated to fixed points of the flow, where scale invariance is realized.

- These fixed points may control the IR behavior of the theories. (example: Wilson-Fisher fixed point) Wilson (1971), Wilson and Fisher (1972)
- Fundamental physics in a QFT description require renormalizability conditions which in the most general case goes under the name of Asymptotic Safety: existence of a fixed point with a finite number of UV attractive directions. Asymptotic freedom is a particular case with a gaussian fixed point.

Weinberg (1979)

Common formulations:

- Perturbation theory in presence of small parameters, e.g. ε-expansion below the critical dimension
- Wilsonian non perturbative, exact equations but not solvable in practice. (Polchinski and Wetterich/Morris equations)

Action description

The main constraints are given by the <u>field content</u> and the symmetries, but this leaves still too many possible theories for a generic dimension d.

It is therefore useful to start from some kind of Landau-Ginzburg description to single out some possible solutions.

$$
S = \int \mathrm{d}^d x \sum_i g_i O_i(\phi)
$$

• This is the starting point for an RG analysis.

Couplings are coordinates in theory space, spanned by a basis of operators

The points corresponding to critical theories may be CFT fixed by the **Conformal data**: the scaling dimensions of the primary operators and the structure constants defining their 3 point correlators. No lagrangian formulation is required.

Universal data and RG expanding the beta functions. If α parameters the deviation from the fixed point α *Mi ^j* ⌘ *∂bⁱ ∂g^j* **TESS** $\overline{ }$ \mathbf{f} ⇤

C˜*a*

bc ⁼ ^X

i,j,k

Sa

ⁱ Nⁱ

In the neighborhood of a fixed point it is convenient to characterize the flow by Taylor by Taylor by Taylor by

jk (*S*1)

^b (*S*1)

^c , (2.9)

(2.4)

ab h*Oc*(*x*)*···*i (2.11)

j (through the second

 $\overline{1}$

^j is already diagonal, or its left and right

⁵ In the RG

^j with the correspond-

How to get in an RG framework informations on the critical theory? If conformal, the so called conformal data? an RG framework informations on the critical theory? Φ corrections agreement data? inhomogeneous transformations of these coecients under general scheme changes as dis-If conformal, the so called conformal data we to get in an RG framewor *ⁱ ^dgⁱ* ⁺ Â *ij ^dgⁱ ^dg^j* ⁺ *^O*(*dg*3), (2.3) How to get in an RG framework information

• For example in the perturbative ε -expansion approximation using the universal beta function coefficients, e.g. in a massless $\overline{\text{MS}}$ scheme the universal beta function coefficients, e.g. in a massless \overline{MS} scheme • For example in the perturbative ε -expansion approximation using where at the linear level we define the linear level we define the stability matrix \mathbf{r}_i expansion *i*ents, e.g. $\frac{1}{2}$ approximation using $\begin{array}{c} \overline{1} \\ 1 \end{array}$ $\overline{}$

> Critical quantities are encoded in the expansion coefficients describing the flow around the scale invariant point: $\beta^{i}(g_*) = 0$ is scale invariant Critical qualities
around the scale *i M i n Mi**describing* **the** *invariant* **point:** $\beta^{i}(g_*) = 0$ re encoded
cariant poi *i*,*j* Ĩ α is the last two (lower) indices. Ermed guarantes are encoura in the expansion coerrients absorting the now versality class and \mathcal{V} conditions that we assume, a relations that we assume, a relations that we assume \mathcal{V}

 $\mathbb{E}[\mathbf{X}]$ is the neighborhood of a fixed point it is convenient to characterize the flow by Taylor by Taylor

$$
\beta^{k}(g_{*} + \delta g) = \sum_{i} M^{k}_{i} \delta g^{i} + \sum_{i,j} N^{k}_{ij} \delta g^{i} \delta g^{j} + O(\delta g^{3})
$$

$$
M^{i}_{j} \equiv \frac{\partial \beta^{i}}{\partial g^{j}}\Big|_{*} \qquad N^{i}_{jk} \equiv \frac{1}{2} \frac{\partial^{2} \beta^{i}}{\partial g^{j} \partial g^{k}}\Big|_{*}
$$

to renormalize a perturbative expansion of the form (2.8) in which the form (2.8) in which the C *Mi j* $\frac{1}{200}$ *∂g^j* \mathbf{C} \mathbf{a} Moving to a diagonal basis in the linear sector ϵ \cdot li

is scale invariant

), we have

bk

which is symmetric in the last two (lower) indices.

$$
\sum_{i,j} S^a{}_i M^i{}_j (S^{-1})^j{}_b = -\theta_a \delta^a{}_b
$$

7 *∂g^j*

Critical exponents allow for a precise definition of the scaling dimensions of the oper-

ators through the relation *qⁱ* = *d* D*i*. Let us introduce the "canonical" dimensions

∂g^j

∂gk

*∂*2*bⁱ*

∂g^k

Ļ **TREE**

 $\frac{1}{3}$

2

1

Ni

cerned with cases in which either the matrix *Mⁱ*

Each scale invariant point of the RG flow is in one to one correspondence with a uni-

jk ⌘

tially devoted to a devoted to a detailed exploration of this link, first within a simple $\mathcal{F}^{(n)}$

ing eigenvalues being (the negative of) the critical exponents *qa*. We will only be con-

next subsection and then, after a short discussion about scheme dependences of the OPE

, α , β , α , β , β

Universal data and RG It will become clear in the practical examples that will follow this subsection that at $\frac{1}{2}$ *differential direct physical direct physical means in the diagonalizing transformation*

jk (*S*1)

^b (*S*1)

C˜*a*

The beta functions can now be written as a now be written as $\mathcal{L}_\mathcal{D}$

d = *d^c* the *C*˜*^a*

d = *d^c* the *C*˜*^a*

tensor *Nⁱ*

d = *d^c* the *C*˜*^a*

bc ⁼ ^X

i,j,k

C˜*a*

Sa

ⁱ Nⁱ

i,j,k

bc ⁼ ^X

inhomogeneous transformations of these coecients under general scheme changes as dis-

jk (*S*1)

ⁱ Nⁱ

Sa

$$
\theta_a = d - \Delta_a \qquad \qquad \tilde{C}^a{}_{bc} = \sum_{i,j,k} S^a{}_i \, N^i{}_{jk} \, (S^{-1})^j{}_b \, (S^{-1})^k{}_c
$$

RG flow seen along the eigendirections around the fixed point up to second of $\frac{1}{2}$ RG flow seen along the eigendirections around the fixed point up to second order $I_{\rm tot}$ is writing that we are used that $f_{\rm test}$ decides the subsection that α α are eigendifections around the fixed point up to second order

$$
S = S_* + \sum_a \mu^{\theta_a} \lambda^a \int d^d x \, \mathcal{O}_a(x) + O(\lambda^2).
$$

$$
\beta^a = -(d - \Delta_a)\lambda^a + \sum_{b,c} \tilde{C}^a{}_{bc} \lambda^b \lambda^c + O(\lambda^3).
$$

Take home message

cussed in subsection 2.3. For the subsection 2.3. For the quantities in (2.9) MS OPERATION 2.3. For the quantities in (2.9) MS OPERATION 2.3. For the quantities in (2.9) MS OPERATION 2.9 MS OPERATION 2.9) MS OPERATION 2.9

one can extract not only the scaling dimensions, but also, reversing an argument from Cardy for a CFT, some OPE coefficients (structure constants) at order $O(\epsilon)$ from Cardy for a CF1, some OPE coefficients (structure constants) at order C $f(x) = \cos x$ extent $x + \ln x$ and $\sin x$ dimensions but also resultations of anomalization be during the leading order and the leading of the leading of the constants of order f $\frac{1}{2}$ Thom care y for a Cr 1, *b*_a *mensions*, but also, reversing an argument *b*,*c*

$$
\langle \mathcal{O}_a(x) \mathcal{O}_b(y) \dots \rangle = \sum_c \frac{1}{|x-y|^{\Delta_a + \Delta_b - \Delta_c}} C^c_{ab} \langle \mathcal{O}_c(x) \dots \rangle
$$

framework, conversely, the knowledge of the beta functions could permit (in principle) the

Linear term coefficients transform homog **Possible**
Quadratic term coefficients transform inhomogeneously
dependence of the coefficients transform inhomogeneously Linear term coefficients transform homogeneously
Possible scheme here [7]. h*Oa*(*x*) *Ob*(*y*)*···*i = Â Quadratic term coefficients transform inhomogeneously Possible scheme

Possible scheme cussed in subsection 2.3. For the quantities in α repetitives in (2.9) MS OPENICE dependence! D*a*+D*b*D*^c* nc *ab* h*Oc*(*x*)*···*i (2.11) dependence!

⁵ In the RG

^c , (2.9)

Deformations are relevant, marginal or irrelevant depending on the value of the related

This formula is the familiar expression for beta functions in \mathbb{F}_p perturbations in \mathbb{F}_p

|x y|

c

^c , (2.9)

Perturbative interlude: Ising Universality Class ε-expansion below d=4 for the LG critical model *L* = 1 2 $(\partial \phi)^2 + g\phi^4$ It will be computed that \mathbb{I} *d* α interlude: Ising Chiversality Class $\overline{O}(n)$ corrections agreement as $\overline{O}(n)$ results for all available comparisons, despite the general available comparisons, designations, despite the general available comparisons, designations, designations, designat w d=4 for the LG chucal moder $\mathcal{L} = \frac{1}{2}(\mathit{U}\psi) + \mathit{Y}\psi$

Leading counterterms in perturbation theory at order g^2 , dim reg $\overline{\text{MS}}$

$$
\mathcal{L}_{c.t.} = \frac{1}{\epsilon} \frac{1}{2(4\pi)^2} (12g)^2 \phi^4 - \frac{1}{\epsilon} \frac{1}{6(4\pi)^4} (4!g)^2 (\partial \phi)^2
$$
\n
$$
12g\phi^2
$$
\n
$$
12g\phi^2
$$
\n
$$
12g\phi^2
$$
\n
$$
12g\phi^2
$$
\n
$$
4!g\phi
$$

i,j,k

Rescaling the coupling: $g \to (4\pi)^2 g$ beta function: $\beta_g = -\epsilon g + 72g^2$

$$
\begin{array}{ccc}\n\bullet & \bullet & \bullet & \bullet & \bullet \\
\text{upling:} & g \to (4\pi)^2 g & \text{Two fixed points:} \\
\text{by} & g = -\epsilon g + 72g^2 & \text{UV gaussian} & \text{IR Wilson-Fisher} \\
\bullet & \bullet & \bullet & \epsilon\n\end{array}
$$

next subsection and then, after a short discussion about scheme dependences of the OPE

cussed in subsection 2.3. For the subsection 2.3. For the quantities in (2.9) MS OPERATOR WE will call the quantities in \mathbb{R}

 $\eta =$

54

Anomalous dimension: $\eta = 2\tilde{\gamma}_1 = 96g^2$ $\eta = \frac{e}{54}$ framework, conversely, the knowledge of the beta functions could permit $\mathbf{v}^{\mathbf{H}}$

 η is a universal quantity, independent from any coupling reparameterization! extractive independent from ony coupling reperpendents of the rest of the rest of the rest of the rest of the t_1 decreases, the and the and the and the and the asimple of the simple example in the simple σ

Functional perturbative RG example: Ising UC 1

How to study deformations around the Wilson-Fisher fixed point? $d = 4 - \epsilon$ (@) $\frac{u}{u} = 4 - \epsilon$ $\overline{}$ ² + *g*1 + *g*2² + *g*3³ + *g*4⁴ (1.1) $\overline{}$ (@) 2 + *g*₂ + *g*

 $\mathcal{L} =$ 1 2 $(\partial \phi)^2 + g_1 \phi + g_2 \phi^2 + g_3 \phi^3 + g_4 \phi^4$ *L* = Couplings: **L** $\overline{2}$ $\overline{1}$

Dimensionful beta functions *z e differential beta ranctions example.* $\beta_3 =$

✏

eS⇤[*L*] =

✏

L =

Sk[] = ¹

⁴ ³²⁶⁴ *^g*³

⁴ = 72 *g*²

 $\beta_3 = 72 g_4 g_3 - 3312 g_3 g_4^2$ $\beta_1 = 12\,g_2g_3 - 108\,g_3^3 - 288\,g_2g_3g_4 + 48\,g_1g_4^2$ $\beta_2 = 24 g_4 g_2 + 18 g_3^2 - 1080 g_3^2 g_4 - 480 g_2 g_4^2$ $\beta_4 = 72 g_4^2 - 3264 g_4^3$ $\beta_1 =$ *Z*()(@) ² + *V* () (1.2) a) $\beta_3 = 72 g_4 g_3 - 3312 g_3 g_4^2$ $\beta_4 = 72 \, q_4^2 - 3264 \, q_4^3$ ³ = 72 *^g*4*g*³ ³³¹² *^g*3*g*² *Z*()(@) ² + *V* () (1.2) ³ = 72 *^g*4*g*³ ³³¹² *^g*3*g*²

^H (*L*) (1.3)

(1.3)

˙⌘ ⇤@⇤ (1.5)

² (*L*)¹

4

4

10 *^D ^µ*⇤ *^eS*[] ¹ *S*˙ ⇤ = 1 $\overline{2}$ Tr ˙ *^H* $\beta_V = \frac{1}{2}n$ *L^L* $S^{(1)} + a^{-1}$ $\frac{V^{(2)}}{2}$ (4π) ² $\frac{2}{b} + b \frac{V^{(2)}}{2}$ $\frac{f(V^{(3)})^2}{(4\pi)^4} + \cdots$ $B_V = \frac{1}{2} \eta \phi V^{(1)} + a \frac{(V^{(2)})^2}{4} + b \frac{V^{(2)}(V^{(3)})^2}{4} + \cdots$ $a =$ $b=-\frac{1}{2}$ $c = -\frac{1}{6}$ *ek*[¯] = *D µ^k e* ¯ *·*(¯)*Sk*[¯] (1.5) $D_{p+1}(1)$ $\qquad \qquad (V^{(2)})^2$ $\qquad \qquad h^{(2)}(V^{(3)})^2$ $\qquad \qquad a=\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ \overline{z} $U^{(4)}$ ² *^H* (*L*) (1.4) ✓ 2*S*⇤ *S*⇤ *^L ^L* At functional level: 1 loop 2 loop 1 2 2 Δt functional level: Δt Δt $\mathcal{L} =$ 1 2 $Z(\phi)(\partial \phi)^2 + V(\phi)$ (in) $\sqrt{2}$ $\frac{1}{2}$ $\$ ³ = 72 *^g*4*g*³ ³³¹² *^g*3*g*² 4 2 loop $c = -\frac{1}{6}$ 6 1 lo $\beta_V =$ 1 2 $\eta\phi V^{(1)}+a$ $(V^{(2)})^2$ $\frac{(4\pi)^2}{(4\pi)^2} + b$ $V^{(2)}(V^{(3)})^2$ $\beta_V = \frac{1}{2} \eta \phi V^{(1)} + a \frac{(V}{(4\pi)^2} + b \frac{(V}{(4\pi)^4} + \cdots)$ 2 $\frac{1}{2}\eta Z + \frac{1}{2}\eta \phi$ 1 \overline{a} $(1) + c \frac{(V^{1}}{V^{2}})$ (y) ² $\frac{1}{4} + \cdots$ (1.5) $c = -\frac{1}{4}$ ✏ *Z* = Z *,* = *^L* + *^H ,* = *^L* + *^H* (1.6) $\overline{1}$ \overline{a} $=\frac{1}{2}\eta\phi V^{(1)}$ $\bigcup_{\pm a}$ (*V* $+\frac{a^2}{(4\pi)^2}$ $\frac{1}{4}V^{(2)}(V)$ $b\frac{1}{(4\pi)^4} + \cdots$ $\beta_Z = \eta Z +$ 1 2 $\eta \phi Z^{(1)} + c$ $(V^{(4)})^2$ $\frac{1}{(4\pi)^4} + \cdots$ $c = -\frac{1}{c}$ ✏ *Z* = Z *D eS*[] ¹ *,* = *^L* + *^H ,* = *^L* + *^H* (1.6) Take home message

FPRG for multicritical models the beginning of the next Section why, with a CFT approach, all the subject approach, all the subject approach or multicritical models and the action to contract the action of the action to contract the action of the acti

Landau-Ginzburg lagrangian $d = d_m$

$$
d=d_m-\epsilon
$$

g must be a purely imaginary number for the odd potentials. We will see in more detail at

$$
S[\phi]=\int d^dx\, \Bigl\{\frac{1}{2}(\partial\phi)^2+\mu^{\left(\frac{m}{2}-1\right)\epsilon}\frac{g}{m!}\phi^m\Bigr\}
$$

^L ⁼ ¹ U_{pper} critical dimension

$$
d_m=\frac{2m}{m-2}
$$

(4⇡)⁴ ⁺ *···* (1.4)

(4⇡)⁴ ⁺ *···* (1.5)

 $\mathcal{L} =$ 1 $Z(\phi)(\partial \phi)^2 + V(\phi)$ ¹ = 12 *^g*2*g*³ ¹⁰⁸ *^g*³ ³ ²⁸⁸ *^g*2*g*3*g*⁴ + 48 *^g*1*g*² 4 in *d* dimensions, for *d* suciently close to the upper critical dimension as in Eqs. (1.2) ² = 24 *g*4*g*² + 18 *g*² ³ ¹⁰⁸⁰ *^g*² ³*g*⁴ ⁴⁸⁰ *^g*2*g*² and α reader shown reader shown reader shown α $\frac{1}{2}$ in comparing in the comparing in the comparing in the One marginal interaction at d_m .

Study of deformations: we limit to a truncation imit to a truncation $\mathcal{L} = \frac{1}{2}Z(\phi)(\partial \phi)^2 + V(\phi)$ $2^{(1)}$. Brucy of acrommatons. We mint to a transaction $2^{(1)}$ controlled by mean-field critical exponents, while below the upper critical dimension the

RG, even RG, even O'Dwyer, Osborn Ann. Phys. 323 (2008) 1859 Codello, Safari, G.P.V., Zanusso EPJ C78 (2018)

Codello, Safari, G.P.V., Zanusso EPJ C78 (2018) 1 *^V* ⁼ ¹ (*V* (2))² *V* (2)(*V* (3))² fluctuations are strong enough to change the scaling properties and to provide the field

⁴ ³²⁶⁴ *^g*³

2

RG, odd RG. odd Codello, Safari, G.P.V., Zanusso Phys. Rev. D98 (2017) 081701 701 ² = 24 *g*4*g*² + 18 *g*² must be tuned to its critical value as will be done later in the paper. Nevertheless, we therefore the spin \mathcal{L}_1 is only one specific reality is only one specific reality is only one specific reality

✏ Multi-loop diagrams at functional level: could exclude the strictly dimension of the strictly dimensional couplings $\mathcal{L}(\mathcal{C})$. α should not generally be confused. The paper with universality confused with universality confused to a greater t $m = 2n$

This involves both the *V* and *Z* functions and contributes to the flow of *Z* for *r* = *n*,

$$
\text{LO:} \qquad \sum_{\substack{v^{(n)} \\ \beta_v = -dv(\varphi) + \frac{d-2+\eta}{2} \varphi v'(\varphi) + \frac{n-1}{n!} \frac{c^{n-1}}{4} v^{(n)}(\varphi)^2}}^{\nu^{(n)}} \qquad \text{NLO:} \qquad \bigotimes_{\substack{z^{(n-1)} \\ \beta_z = -\eta z(\varphi) + \frac{d-2+\eta}{2} \varphi z'(\varphi) - \frac{(n-1)^2}{2} \frac{c^{2n-2}}{4} v^{(2n)}(\varphi)^2}}^{\nu^{(n)}} - \frac{n-1}{4} c^{2n-2} \Gamma(\delta_n) \sum_{\substack{r+s+t=2n \\ r,s,t \neq n}}^{\beta_{st}} \frac{k_{rst}^n}{v^{(r+s)}(\varphi) v^{(s+t)}(\varphi) v^{(t+r)}(\varphi)}^{\nu^{(s+t)}} \qquad \text{Rescaling functions and fields to\n-\frac{(n-1)^2}{16n!} c^{2n-2} \sum_{s+t=n}^{\infty} \frac{n-1+L_{st}^n}{s!t!} v^{(n)}(\varphi) v^{(n+s)}(\varphi) v^{(n+t)}(\varphi)}^{\nu^{(s+t)}} \qquad \text{Rescaling functions and fields to\n\text{dimensionless quantities } v(\varphi), z(\varphi)
$$

where the integers *r*,*s*, *t* are implicitly taken to be positive, and the quantities *Kⁿ*

stream and defined as follows

ns

rst ⁼ ^G

 $V^{(r+s)}$ ⁴ = 72 *g*² V^s)
|} Z *D eS*[] ¹ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ $V^{(t)}$ r /// // \\\\\\ s t $V^{(r+t)}$ $V^{(s+t)}$ $\frac{s}{V^{(s+t)}}$ $V^{(s)}$ t $V_{c.t.}^{(r)}$ r $V(r)$

$$
\beta_z = \eta z(\varphi) + \frac{d-2+\eta}{2} \varphi z'(\varphi) - \frac{(n-1)^2}{(2n)!} \frac{c^{2n-2}}{4} v^{(2n)}(\varphi)^2 + \frac{n-1}{n!} \frac{c^{n-1}}{2} \left[z^{(n)}(\varphi) v^{(n)}(\varphi) + z^{(n-1)}(\varphi) v^{(n+1)}(\varphi) \right]
$$

Rescaling functions and fields to

^D ^µ⇤ *^eS*[] ¹

 S⇤ *^L* *S*⇤

² (*L*)¹

˙ *^H*¹

^µ˙ ⇤

 $\overline{}$

1
1940

^H (*L*) (1.6)

11 which will therefore be of $\mathcal{O}_\mathcal{A}$ and there are the are three different di agrams of this kind depending on whether one, two or none of the fields in (*∂f*)² are real dimension *d* = *d^m* ✏. Theories living in continuous dimensions have already been

S˙

ِ اللہ عليہ
مسجد

tra
Trans

 $V^{(r+s)}$

˙ *^H*

✓ 2*S*⇤

FPRG for multicritical models mension the spectrum of the theory is almost Gaussian, we can infer that the couplings \mathcal{C} *FPRG* for multicritical m *mix with any other coupling.* Staring from $f(2)$ side and only couplings of operators in the same column mix together. If we arrange the couplings of multicritical models of the increasing order of the increasing order of the increasing order o furthermore, we sort them for increasing order of derivatives of their corresponding

General pattern of mixing of the operators present in the truncation with the *^O*(*∂*2) couplings of (*∂f*)2, *···* , *^f*2*n*3(*∂f*)2. From *^f*4*n*2, *^f*2*n*2(*∂f*)² the *^O*(*∂*4) **EX** General pattern of mixing of the operators present in the truncation

$$
V: \t 1 \phi \cdots \phi^{2n-1} \phi^{2n} \cdots \phi^{4n-3} \phi^{4n-2} \cdots
$$

\n
$$
Z: \t (\partial \phi)^2 \cdots \phi^{2n-3} (\partial \phi)^2 \phi^{2n-2} (\partial \phi)^2 \cdots
$$

\n
$$
W_1: \t \phi \Box^2 \phi \cdots
$$

\n
$$
W_2: \t (\partial_{\mu} \partial_{\nu} \phi)^2 \cdots
$$

\n
$$
W_3: \t (\Box \phi)^2
$$

Wa(*f*) and those of the higher derivative operators, in powers of the field, starting with *f*⁰ = 1. Using dimensional analysis and recallling that close to the upper critical di-

where each row collection the operators included in the function shown on the function shown on the left-hand side and and and only composite operators in the same composite operators in the same column mix to same control of \mathbb{R}^n In sumplays dimensions of composite energters ments, and for *i* 0 Anomalous dimensions of composite operators

$$
\tilde{\gamma}_i = \frac{2(n-1)n!}{(2n)!} \frac{i!}{(i-n)!} \epsilon \qquad \tilde{\omega}_i = \frac{2(n-1)n!}{(2n)!} \frac{(i+1)!}{(i-n+1)!} \epsilon \qquad \text{Stability matrix } M
$$
\nis triangular.

At leading order the
\n
$$
\tilde{\gamma}_i = \frac{2(n-1)n!}{(2n)!} \frac{i!}{(i-n)!} \epsilon
$$
\n
$$
\tilde{\omega}_i = \frac{2(n-1)n!}{(2n)!} \frac{(i+1)!}{(i-n+1)!} \epsilon
$$
\nStability matrix M
\nis triangular.

M(2)

 $M^{(4)}$

...

1

 $\overline{}$

dLdL + *···* (4.4)

where each row collection the operators included in the function shown on the function shown on the left-hand

 $\sqrt{2}$

 $M^{(0)}$

 $\overline{}$

$$
\tilde{\gamma}_i = i\frac{\eta}{2} + \frac{(n-1)i!}{(i-n)!} \frac{2 \, n!}{(2n)!} \left[\epsilon - \frac{n}{n-1} \, \eta \right] + 2n \, \eta \, \delta_i^{2n} \n+ \frac{(n-1)i!n!^6}{(2n)!^2} \Gamma(\delta_n) \sum_{\substack{r+s+t=2n \\ r,s,t \neq n}} \frac{K_{rst}^n}{(r!s!t!)^2} \left[\frac{2n!}{3(i-n)!} - \frac{r!}{(i-2n+r)!} \right] \epsilon^2 \n+ \frac{(n-1)^2 i!n!^5}{(2n)!^2} \sum_{s+t=n} \frac{n-1+L_{st}^n}{(s!t!)^2} \left[\frac{1}{(i-n)!} - \frac{2s!}{n!(i-2n+s)!} \right] \epsilon^2.
$$

dLdL

from the term proportional to *h* in (5.4), does not contribute in the relevant sector. How-

į *L*⇤

mal invariance. Several non trivial informations on the critical theory can then be extracted by probing

18

dL

work.

anomalous parts¹⁰

l *L*⇤

arbitrary of \overline{OPE} coefficients are read off the quadrat **d**2^b For the relevant components are read off the quadratic expansion of the **OPE coefficients are read off the quadratic expansion of the beta functions**

Other studies

• Multicritical higher derivative theories: there can be many marginal operators at criticality, results still to be understood in CFT.

Safari, G.P.V., Phys. Rev. D98 (2017) 081701, EPJ C78 (2018) 251

• Shift symmetric theories

Safari, G.P.V. in preparation

Multifield theories

• Potts models (cubic) Potts models (quintic)

Osborn, Stergiou arXiv:1707.06165 Codello, Safari, G.P.V., Zanusso in preparation

Non trivial in $d=3$

Perturbative ε-expansion useful guide towards non perturbative regimes.

Non perturbative functional RG flows and try to understand how to obtain them from a more formal point of view. The competition continues for the most general form, which is the most general form, which is a continued for most general form, which is a continued for the most general form, which is a continued for the most general for \mathcal{L} and \mathcal{L} are general proper time regularization leading to the family of wilsonian flows of wilsonian flows \mathcal{L} won perturbative runctional KG TIO *^d*⇤*eS*⇤['] ⁼ '(*x*) [*d*']*eS*⇤['] is manifestly indepedendent

⇤ *d*

Perturbation theory is very powerful to derive some qualitative informations even for infinite set of universal data, but for strongly interacting theories **2** non perturbative tools are needed. ation theory is very powerien to active some quantum ve informations

• Wilsonian flows: to another wilsonian action *S*⇤⁰ . One can derive the following relation

for some *^x*['], so that the partition function *Z* = R

 require the partition function to be independent from a UV cutoff. In general one can have $\frac{1}{2}$ and the independent from In general one can have $Z = \int$ $[d\varphi]e^{-S_\Lambda[\varphi]}$ is manifest The coarse-contribution function in the independent from a UV cutoff general form and action in the most general form and UV cutoff **z**
*z
<i>x***</sub>** e independent from a UV cute V **cuton.**
 $I = S_{\lambda}[\omega]$

$$
\Lambda \frac{d}{d\Lambda} e^{-S_{\Lambda}[\varphi]} = \int\!dx \frac{\delta}{\delta\varphi(x)} \left(\psi_x^{\Lambda}[\varphi] e^{-S_{\Lambda}[\varphi]} \right)
$$

order to realize the coarse-graining associated to the coarse-graining associated to the flow in Eq. (I.11), as a particular realization of the flow in Eq. (I.11), as a particular realization of the flow in Eq. (I.11), as

$$
\Lambda \frac{d}{d\Lambda} S_{\Lambda}[\varphi] = \int dx \left(\frac{\delta S_{\Lambda}[\varphi]}{\delta \varphi(x)} \psi_x^{\Lambda}[\varphi] - \frac{\delta \psi_x^{\Lambda}[\varphi]}{\delta \varphi(x)} \right)
$$

^d⇤ *^eS*⇤⁰ ['0]

, (IV.3)

. (IV.4)

the coarse-graining scheme of the kind '(*x*) = *b*⇤['0](*x*), where '⁰ is associated to the scale ⇤⁰

dx

⇣

⇤

^x [']*eS*⇤[']

⌘

[*d*']*eS*⇤['] is manifestly indepedendent

 $\frac{1}{2}$ and $\frac{1}{2}$ an $\frac{1}{2}$ -graining corresponds to a non trivial action-dependent field redefinition In general the flow induced by coarse-graining In conoral the flow induced by coarse graining ⇤ *^x* ['] = *eS*⇤['] to a non trivial action-dependen $\delta \Lambda_{ab} \Lambda_{bc}$, (IV.3)

of the more general flow equation, related to Eq. (IV.2),

$$
\varphi'(x)=\varphi(x)-\tfrac{\delta\Lambda}{\Lambda}\psi^\Lambda_x[\varphi]
$$

Wilson-Polchinski RG flows *^Z*[*J*] = ^Z *^Z*[*J*] = ^Z /*I|SON-* $\overline{1}$ ¹/¹/³ \overline{K} ¹ \overline{N} ⇤⁰ []+*J·*' (1.1) *^eW*⇤['*L,J*] ⁼ *^Z*⇤['*L, J*] = *^e* ¹ *' HINSKI KG HOWS*

1

2'*·*1*·*'*S^I*

[*d*'] *e*

^Z⇤⁰ [*J*] = ^Z

$$
Z_{\Lambda_0}[J] = \int d\varphi] e^{-\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi - S_{\Lambda_0}^I[\varphi] + J \cdot \varphi}
$$

Split in low (L) and $\varphi = \varphi_L + \varphi_H$ $\Delta = \Delta_L + \Delta_H$ $\mu_{\text{S}}(1)$ chergy modes $\mu_{\text{S}}(1)$ $\mu_{\text{S}}(1)$ high (H) energy modes $\Delta = \Delta_I$ [*d*'] *e* $(0 - \omega t + \omega t)$ $\mathcal{L} + \Delta H$ $\frac{d}{d\lambda_0}[\varphi]+J\cdot\varphi$ **Spin III IOW** (**L**) and **Split in low (1.) and** $\varphi = \varphi_L + \varphi_L$

1

4

[*d*'] *e*

⇤⁰ [']+*J·*' (1.1)

2'*·*1*·*'*S^I*

² *^J·H·J*+*J·*'*LS^I*

2'*·*1*·*'*S^I*

^r²) + *^ZO*(

@*tR^k*

@*tR^k*

^r²) + *^ZO*(

⌘¹

₩

2

^µ˙ *^k*

^µ˙ *^k*

†

@⌧ ↵

@⌧ ↵

\$

†

1

@⌧ ↵

Tr ^h

1

˙ *^H*¹

˙ *^H*¹

² (*L*)¹

H

˙⌘ ⇤@⇤ (1.5)

Tr ^h

˙⌘ ⇤@⇤ (1.8)

' = '*^L* + '*^H* (1.2)

˙ *^H*¹

¯ *·*(¯)*Sk*[¯] (1.6)

¯ *·*(¯)*Sk*[¯] (1.9)

˙⌘ ⇤@⇤ (1.5)

˙⌘ ⇤@⇤ (1.7)

¯ *·*(¯)*Sk*[¯] (1.6)

¯ *·*(¯)*Sk*[¯] (1.8)

⇤⁰ []+*J·*' (1.1)

⇤⁰ []+*J·*' (1.1)

= *^L* + *^H* (1.3)

^H (*L*) (1.5)

⇤[*H·J*+'*L*] (1.4)

^H (*L*) (1.4)

^H (*L*) (1.4)

^H (*L*) (1.5)

 $p < \Lambda$ $\frac{1}{2}$ IOT $|p| < \Lambda$ *<u>ghly</u> for* φ *L* has support roughly for $|p| < \Lambda$

2

2

k[] = extr *^J* (*^J · ^Wk*[*J*]) *Sk*[] (1.11)

@*t^k* ⁼ ¹

@*t^k* ⁼ ¹

Tr⇣

Tr⇣

 $\overline{2}$

(2)

µ⇤

¯ *·*(¯)*Sk*[¯] (1.7)

^k + *R^k*

^k + *R^k*

⇤[*H·J*+'*L*] (1.4)

 $\frac{1}{2}$ the *s* \overline{c} $\sum_{k=1}^{\infty}$ *S*¹ from racting action S^1 from modes one defines the interacting action S^I from Integrating the high energy modes one defines the interacting action S^I from = *^L* + *^H* (1.3) $\ddot{}$ ines 2 he in^o ✓ 2*S*⇤ ι *d* ι ι z acti *^L* $\overline{\mathbf{u}}$ S^I_Λ from \mathbf{m} $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$

$$
e^{W_{\Lambda}[\varphi_L,J]} = Z_{\Lambda}[\varphi_L,J] = e^{-\frac{1}{2}J\cdot\Delta_H\cdot J + J\cdot\varphi_L - S^I_{\Lambda}[\Delta_H\cdot J + \varphi_L]}
$$

Sk[] = ¹ ²*·R^k ·* according Φ **Polchinski** equation *^H* (*L*) (1.5) *eS*⇤[*L*] = $\overline{}$ **d 1** (*C*)²*l***</sup> 1 (***C***)²***l*** 1 (***C***)²** *^H* (*L*) (1.5) It is flowing according to the Polchinski equation *E*^{*x*} *is flowing executing to the Polebingki equation* **S** to the **F OICIII** *SI* zquation

$$
\Lambda \frac{d}{d\Lambda} S^I_{\Lambda} [\varphi] = \frac{1}{2} \int dx dy \, \left(- \Lambda \frac{d}{d\Lambda} \Delta_H \right)_{xy} \left[\frac{\delta S^I [\varphi]}{\delta \varphi(y)} \frac{\delta S^I [\varphi]}{\delta \varphi(x)} - \frac{\delta^2 S^I [\varphi]}{\delta \varphi(y) \delta \varphi(x)} \right] + \text{const}
$$

 $ext{from the IV}$ $\frac{1}{2}$ **h** $\frac{1}{2}$ *DV* cutof ˙⌘ ⇤@⇤ (1.6) $\overline{1}$ *P*_{*P*} *P*_{*P*} *<u>R</u>* $\frac{1}{2}$ *<i>p* $\frac{1}{2}$ *p* $\frac{1}{2}$ *p* $\frac{1}{2}$ *p* $\frac{1}{2}$ *p* $\frac{1}{2}$ *<i>p* $\frac{1}{2}$ $\frac{1}{2}$ **R**
*R***k**(*p*2) **II** μ **C** (*p*²) The partition function is independent from the UV cutoff ²

◆ + const

L^L

ek[¯] =

S^I

⇤

S^I ⇤

^L

2

⇤

✓ 2*S^I*

S˙

 \mathbf{z}

1

Tr

˙ *^H*

Sk[] = ¹

^L

^Rk(*p*2) *>* 0 for *^p*² ⌧ *^k*²

^L

1PI effective average action RG flow $P|_{\epsilon}$ \overline{a} H P P H \hat{H} A B B B C ^{A} B ˙⌘ ⇤@⇤ (1.10) ✓ 2*S^I* ⇤ *S^I* ⇤ c t ive ave iver dge ✓ 2*S*⇤ *S*⇤ *S*⇤ ◆

www.communications.com

⇤ *^d*

^d⇤*^H*

xy

'(*y*)

dxdy ✓

$$
e^{-W_k[J]}=Z_k[J]=e^{-\Delta S_k[\frac{\delta}{\delta J}]}Z_k[J]=\int [d\varphi]\;e^{-S[\varphi]-\Delta S_k[\varphi]+J\cdot\varphi}
$$

्तर अन्तर अस्त्री संस्कृति ।
अस्ति ।

2

Infrared regulator: $\Delta S_k[\varphi] = \frac{1}{2}\varphi \cdot R_k \cdot \varphi$ ✓ 2*S*⇤ *L*_L ed re re *S*⇤ *^L* r: $\Delta S_k[\varphi] =$ $\frac{1}{2}\varphi \cdot R\iota$ *INITATEG regulator:* $\Delta S_k[\varphi] = \frac{1}{2}\varphi \cdot R_k$

('*^c*) *· ^H ·* ('*^c*) = *^S^I*

⇤ *d*

d⇤*S^I*

$$
R_k(p^2) > 0 \text{ for } p^2 \ll k^2
$$

$$
| = \frac{1}{2}\varphi \cdot R_k \cdot \varphi
$$

$$
R_k(p^2) \to 0 \text{ for } p^2 \gg k^2
$$

$$
R_k(p^2) \to \infty \text{ for } k \to \Lambda \; (\to \infty)
$$

\$

'(*y*)'(*x*)

˙ *^H*¹

'(*x*) 2*S^I* [']

k[] = extr *^J* (*^J · ^Wk*[*J*]) *Sk*[] (1.11)

1

d*Dx* d⌧

◆

Tr⇣

Zk()(@)

¯ *·*(¯)*Sk*[¯] (1.13)

^d*x*¹ *···* ^d*xn*(*n*)

^Vk() + ¹

^k() = *d v*˜*^k* +

✓*d*

d*dx*

v˜˙

2

2

^Rk(*p*2) ! 1 for *^k* ! ⇤ (! 1)

D

^µ˙ ⇤

^eWk[*J*] ⁼ *^Zk*[*J*] = *^eSk*[

¯ *·*(¯)*Sk*[¯] (1.11)

^H (*L*) (1.5)

 $\frac{1}{\sqrt{2\pi}}$ Legendre transform Legendre transform

Sk[] = ¹

2

 $\ddot{}$

²*·R^k ·*

⇤['*^c*

 $+ 1/2$

$$
\Gamma_k[\phi] = \operatorname{extr}_{J}(J \cdot \phi - W_k[J]) - \Delta S_k[\phi] \qquad \qquad e^{-\mathbf{1}_k[\phi]} = \int [d\varphi] \ e
$$

$$
V_k[J]) - \Delta S_k[\phi] \qquad e^{-\Gamma_k[\phi]} = \int [d\varphi] \ e^{-S[\varphi] + \frac{\delta \Gamma_k}{\delta \phi} \cdot (\varphi - \phi) - \Delta S_k[\varphi - \phi]}
$$

|p| < ⇤ (1.7)

 $\mathcal{N} \rightarrow \mathcal{N}$

= *^L* + *^H* (1.3)

\$

s equation Wetterich/Morris equation *^Rk*(*p*2) ! 0 for *^p*² *^k*² **etterich/Morris equation**

⇤ =

Tr

n!

n

˙ *^H*

✓ 2*S^I*

$$
\partial_t \Gamma_k = \frac{1}{2} \operatorname{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right] \qquad t = \ln k / k_0
$$

1
1

 $\Gamma_\Lambda[\varphi^c] + \frac{1}{2}$ 2 $(\varphi^c - \Phi) \cdot \Delta_H \cdot (\varphi^c - \Phi) = S^I_\Lambda[\Phi]$ $\frac{1}{2}$ *^Rk*(*p*2)=(*k*2*p*2)✓(*k*2*p*2) (1.13) Legendre type relation between Wilsonian action and effective average action *^Vk*() + ¹ *Zk*()(@) ² + 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 - 2022 $\overline{1}$ ian action an 1 effective average *k*[] = ^X 1 Z $\frac{1}{2}$

⇤

 $\frac{17}{2}$ Multiple Stalle Library at the Reserve theories Consider a Quency of the consideration and produced the constant of the produced symmetries. The constant of the consta LAR ORTHUCTURE QFT of one real scalar field and Dirac fermions *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define invariance over a the 2-3 systems lawed to standard Yukawhar by *H*() = *y* which are odd under *Z*² then one requires that spinor transform as ! *i* and ¯ ! *ⁱ* ¯. A generalization of local interactions with such a symmetry then requires an odd *k* ⇥ *, ,* ¯ ⇤ = **X***fb, W*_f*I* with the simplificial for the possibility to the possibility to the possibility to the possibility of the possibility of ϕ transformation, which would require an even function *H*(). In this work we shall consider the case of an odd Yukawa potential *H*. WE SHALL MAKE OUR ANALYSIS IN THE LATTER OF THE LATTER OF THE LATTER OF THE DERIVATION (LOWEST ORDER OF THE DE sion, with *Z* = *Z* = 1 and therefore zero anomalous dimensions) (Not sure which is the want to write more in general... **production in the LPA' (including a dependence in the anomalous)** After rescaling to dimensionless variables, the flow equation for the two potentials are given **by vise + draw** *d* 2 + ⌘ ² *^v* ⁰ + *C^d* ¹ ⌘ *d*+2 1 + *^v*⁰⁰ *^X^f* **h an Australian and Bandard and HOVEL LIL**
MGCL COBUL *C^d* 3 **hO** 2 **h** 2 *h* 2 (1 + *h*2) ² (1 + *v*00) + ¹ ⌘ *d*+2 (1 + *h*2) (1 + *v*00) where we have defined the constant of the const
where we have defined the constant of the seco **d** = (4ª) giving the scaling solutions in the coupled system of the coupled system of two coupled sy **ordinary, it meet het verking four** 0 = *dv* + 1962 ² *^v* $\frac{1}{2}$ 1999 1 + *^v*⁰⁰ *^X^f* 1 **h** 2 **Symmetries: In first symmetries: The symmetries of the symmetries which is the second of the symmetries of the** id
La **da da da** 大好 ea
Li *Z*
ZZ Z Z <i>Z <i>Z \mathbf{Z}^T systems \mathbf{Z}^T such that \mathbf{Z}^T is \mathbf{Z}^T . The standard \mathbf{Z}^T is \mathbf{Z}^T in \mathbf{Z}^T is a such that \mathbf{Z}^T is \mathbf{Z}^T is a such that \mathbf{Z}^T is \mathbf{Z}^T is a such that \mathbf{Z} system which are parametrized by the parametrized by *Hopper Liberty are one requires* the parameter $\frac{1}{2}$ spinor transform as ! *ⁱ* and ¯ ! *ⁱ* ¯. A generalization of local interactions with such a symmetry then requires an odd **H** *H*_{*f*} D₁ A_f D₁ A dimension dimensional dimensions and we define the process of the possibility of the poss have unchanged spinors under the transformation, which would require an even function *H*(). In this work we shall consider the case of an odd Yukawa potential *H*. WE SHALL MAKE OUR ANALYSIS IN THE LAND MAKE OUR APPROXIMATION (LOWEST ORDER OF THE DERIVATIVE EXPANDING THE DE **seme Diversion, with a zero and consistent in an cost (Not sure we want to write more in general... nevertheless the flow equation for the flow equation for the two** potentials in the LPA' (including a dependence in the anomalous dimensions) of the anomalous dimensions (including a dependence in the anomalous dimensions) of the anomalous dimensions of the anomalous dimensions of the an After rescaling to dimensionless variables, the flow equation for the two potentials are given **ya** *v*˙ = *dv* + *d* 2 + ⌘ **20121121315**
201312121214 **hdhh**abted in *d* 2 + ⌘ ² *^h*⁰ ⁺ $\frac{1}{2}$ <u>|}</u> 42*h* יין
וי *h*0 凞 12 995 2001 *d*+1 k and k ∞ k ∞ k ∞ k ∞ k ∞ \overline{a} *Z*₄ *k* @*Y*₄(*Q_u*) + *z H_k</sub> off marrie contain we choose the* First symmetry: one may reflect which esign symmetry requiring the consideration of the constitution of the magnetic symmetry of the consideration of the consideration of the consideration of the consideration of the mater **Z2 symmetry requirements were use the invariance of the individual to standard to standard the industrial to the** system which are parametrized by H() and standing which are the parametrized by H() and the second under a local \mathcal{G} , the lagrand manufacture of \mathcal{G} and \mathcal{G} is and \mathcal{G} with \mathcal{G} and \mathcal{G} the lagrangian \mathcal{G} , \mathcal a symmetry then requires a symmetry then the symmetry and the area fermions in a representation of \mathcal{H} . The \mathcal{H} dimension of the possible \mathcal{P} and \mathcal{P} \mathcal{P} and \mathcal{P} \mathcal{P} and \mathcal{P} have recept spinors under the transformation of the transformation, which was a which would require an even function In the construction of a shall consider the consider the consideration of an opening the consideration of an opening the construction of an opening the construction of an opening the construction of an opening the construc WE SHALL MAKE OUR SHALL MAKE OUR ANDERS IN THE LANGUARD TO THE LANGUARD CONTROL CONTROLLED IN THE DESIGN OF THE sion, with a construction of the construct (Not sure we want to write more in the flow experience of the flow equation for the flow equation for the flow potential in the LPA' (including a dependence in the anomal including a dependence in the anomalous dimensions
(in the LPA' (in the anomalous dimensions) in the anomalous dimensions in the anomalous and the anomalous the PERSULTER RUS TO DIMENSIONLESS VARIABLES VARIABLES VARIABLES VARIABLES VARIABLES, TA THE CLOSE COTS DECLET, TH
P.S. TALLABLES VARIABLES, THE TURNER IN THE TWO POTENTIALS ARE GIVEN TO DIGITAL COLORED CONTINUES CONTINUES A **fit de artigio 2 versel v** ¹ ⌘ *d*+2 15 VONES DUCA ¹ ⌘ *d*+1 1 **http://def** ! *^h*˙ = (⌘ 1) *^h* ⁺ *d* 2 + ⌘ ² *^h*⁰ ⁺ *C^d* $\frac{1}{2}$ 42*h* $\sqrt{\ }$ **h**
196 $\frac{1}{2}$ $\frac{1}{1}$ $W²$ **QV TYPE OF 1.1590** ² (1 + *v*00) $\frac{1}{2}$ **DET 200** *d*+2 1000. UEUNS ALS 6 7 5 6 0 m i d 2 \mathbf{R} 鶨 *^h*⁰⁰ ⇣ 17255 pert **VAIL 4 4 0 MAI** 1 and a polynomial form of the scalar polynomial is used. La Korea, Scherer 2010 (de 4) , Rosa, Vitale, Paris 2011 (d. 3) , 21 We Girl and the The truncation we consider *k* ⇥ $\frac{1}{2}$ First symmetry: *U*(*N^f*) is assumed. There is a special symmetry one may consider which is the **Z2 symmetry funding the invariance of the in** system which are parametrized by *H*() = *y* which are odd under *Z*² then one requires that **spinor transformation as it is and the reduced interaction as it is a general interaction of local interaction** a symmetry then requires an odd *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define *X^f* = *N^f d* for simplicity. There is also the possibility to have unchanged spinors under the transformation, which would require an even function *H*(). In this work we shall consider the case of an odd Yukawa potential *H*. WE SHALL MAKE OUR ANALYSIS IN THE LAND sion, with an anonyment of *Z* 2011. **The sure we want to write more in general.** For a SUSY model both the scalar potential and Yukawa interactions in the on shell lagrangian density in the o
The state of the only lagrangian density in the only lagrangian density in the only lagrangian density in the o **PENCOLLE AREAR WAS SEPTED TO DEPENDENT ON THE SAME FUNCTIONS.**
A studied at symmetry fone may consider which is th Synator Braun, $t_{\rm eff}$ that the West-Zumino model is the West-Zumino model in the finite state of μ the production of the π -For system in Keeps standard Strawher $\frac{1}{\kappa}\sum_{\mu}^{K} \frac{1}{\mu} \sum_{\nu}^{K} \frac{1}{\mu} \sum_{\nu}^{K$ Sections of the Stefan-Boltzmann and the Stefan-Boltzmann land the constitutions of high temperatures the fermion of the fermion do alguna to the flow to the flow to the flow equation of the flow the flow to the flow have been the serve and the other hand we observe the orient we observe the orient of the orient of the orient dia. Synimethy didection irestant hessing to bar wrot who model $\dim \mathbf{G}$ to the presence of a thermal $X_f = N_f \, d_\alpha$ we show in Sect. 2 de secto de Xultawa 'potential de .
Décembre : la casilla in our Romanie de La contrat de In a similar way of the similar way in Control Backston of Nicolas Cape (2011) and Mixis in the U.P. we compute the phase diagram for the phase diagram for the a la Marita Record at finite temperature in Sect. In Sec III. THE N EXTERN HOME IN THE NEW YORK OF THE THE THE DIMENSIONS AT THE LEADER ON THE LARGE DIMENSIONS AT THE LARGE The state are many works on the supersymmetric Wess-Zumino models in both four and two space-time dimensions. Actual mension of the two-dimensions. Actually the two-dimensional model with N ESSIS SON FRACTIS IN PRESSUST THE TURBING TO CAMINAGE COMPACTIFIC cation of the four-dimensional Text of the four-dimensional Text of the threedimensional model with N α is a 1 supersymmetry of the 1 supersymmetry, or the 1 supersymmetry, or the 1 supersymmetry, or the 1 supersymmetry, α is a 1 supersymmetry, or the 1 supersymmetry, α is a 1 supersymmetr other hand, cannot by dimensional reduction by dimensional reduction by dimensional reduction of the cannot be of a local field to a local field theory in four dimensions. Thus it may be a local field of the orient of the
In the call field the company between the company of the company of the company of the company of the company useful to requall the construction of the construction of the the three-dimensional model starting from the real superfield from the real φ, εγκισμη εισφαιλική περιματική **2 ACQUE FOR FORDING** with real (pseudo)scalar fields quadratic pseudo) scalar fields quadratic quadratic field war en de supersymmetry variations are generated by the supersymmetry of the supersymmetry of the supers the supercharge
δηριοφής sports them power two?
ΕτθΕ = σχέδιο σχέδιο του που μετά ∂€₹ (γΩΩ)∂Ω − (γ) − (γ) − (γ)
Γαλίξζε για σρολογίζε We use the metric tensor (ηµ^ν) = diag(1, −1−1) to lower $\frac{1}{1000}$ $\frac{1}{1000}$ the following the following the following the following the following $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{1000}$ $\frac{1}{10$ interactions (not construct the constructions of the constructions of the construction of the construc WE GO BETTLEMENT THE TYPIC SETTLEMENT GIVEN BY A LAGRANGIAN *L*= $\frac{1}{2}$ 20 15 Up
@uffbeliov
P4 14 MM The transport we roofisider is it! **y by by by the file**)
ま **ext** 94
a **da 4x** ₹2 $\mathbf{2}$ Z First symmetry: The symmetry of the symmetry o
The symptom of the symmetry of Z2 systems required the invariance of the system which are parametrized by a y which are one requires the state of the parametrized by $\frac{1}{2}$ spitch as the spinor transform as interactions with such as interactions with such as interactions with such a
The discussion of local interactions with such such such such as interactions with such as interactions with su a symmetry then requires an odd *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define *X^f* = *N^f d* for simplicity. There is also the possibility to have unchanged spinors under the transformation, which would require an even function *H*(). In this work we shall consider the case of an odd Yukawa potential *H*. and estimally limiting ourself to the LPA (Z's=1) and the LPA (Z's=1) and the LPA (Z's=1) and the LPA (Z's=1) I. FORMALIS The truncation we consider the following: *k* ⇥ *, ,* ¯ ⇤ $\frac{1}{10}$ $\frac{1}{2}$ d4*x* **1524** 2 *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* ¯ *^µi*@*^µ* + *i Hk*() ¯ There is a special symmetry one may consider which is the invariance over !!! . For systems linked to standard yukawa by *Hall are one of the spinor are one requires the spinor transform as in the spinor transform as in the spinor transformation of the spinor transformation and the spinor transformation as in the spinor transformation of* of the component of the co
In 15 digital interactions and requires and one component of its instruction of the component of the component *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define X *for* f *defined significity to the phonon spinors in the property to have unchanged spinors under the property of* \mathcal{X} transformation, which we shall consider the vent function $\mathcal{H}(\mathcal{U})$ and $\mathcal{H}(\mathcal{U})$ case of an odd Yukawa potential ion of duneas on i duneas on a sell in the serve of potential of the server of the
Languary cases of the restaurance of the server of the what he case of the case of the Vulsiva State of t WE SHALL MAKE OUR COURT AND THE LANGUAGE OF LANGUAGE OUR DESCRIPTION (LOWEST sion, with *Z* = *Z* = 1 and therefore zero anomalous dimensions) (Not sure we want to write more in general... nevertheless the flow equation for the two potentials in the LPA' (including a dependence in the anomalous dimensions) in the anomalous dimensions) After rescaling to dimensionless variables, the flow equation for the two potentials are given **by** *vdv* $\frac{1}{2}$ + $\frac{1}{2}$ + $\frac{1}{2}$ ² *^v* ⁰ + *C^d* NG SIMBA *d*+2 *<u>f</u> UDNS*
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ALSO CONSIDERS MORE GENERAL YUKAWA INTERACTIONS ALSO CONSIDERS MORE GENERAL YUKAWA INTERACTIONS ALSO CONSIDER **h**an arcole by the first particle of the construction of the **ACA SOMODICAL SHIRONG ARCILISHED AND LINE FOR FIRE ARE ARREST ON A REPRESENTATION** OF DIRACTOR UPPER TO A LINE $T_{\rm H}$ $T_{\rm CO}$ $T_{\rm H}$ and $T_{\rm H}$ summatrices $T_{\rm H}$ is the $T_{\rm H}$ summatrix $T_{\rm H}$ is the $T_{\rm H}$ g mperature ingulati \mathbb{Z} assyformetra ophasinb ∂A r $\partial \partial s$, si $2\partial i$ ba 2 ly We to barametrized to $\partial \partial s$ **H** a *y* which are one of the spinor transformation of the spinor transformation of the spinor transformation as the spinor transformation of the spinor transformation of the spinor transformation of the spinor transforma our to the experiment of the interaction of the second of the symmetry with such a symmetry of the symmetry of *XNe* at the magnetic formulation of the possibility of the possibil the transformation of the consideration of the constant $\mathcal{L}_\mathcal{A}$ **case of an odd Yukawa potential Andrew Portential Andrew Portential Andrew Portential Andrew Portential Portential A** Wernd HAC ROOT BILL MARKER CUTTUR IN THE LIST CONTRACTOR CONTRACTOR TO THE DEPTH OF THE DEPTH OF THE DERIVATIO
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C **. The fixed points of the fixed points** giving the scaling solutions in the LPA are determined by solving the coupled system of two ⁰ + *C^d* ^Φ(x, ^α) = ^φ(x)+¯αψ(x) + ¹ After rescaling to dimensionless variables, the flow equation for the two potentials are given $\overline{d} \psi_{\rm{v}} = \overline{c} \overline{v}$ by \overline{c} and \overline{c} $\text{field }\psi.$ The supersymmetry variations \mathcal{H} with \mathcal{H} \mathcal{H} and \mathcal{H} **de**
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World of Local Philated in the moth such a symmetry *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define **X***f a* **for simplicity of the possibility of the** transformation, which wild require and all consider the strans would require the processive sequence of the strange o case of an odd Yukawa potential *H*. We shall Charle Cup analysis in the LPA sion, with *Z* = *Z* = 1 and therefore zero anomalous dimensions) **(Not sure we want to write more in general...**. nevertheless the two wants for the two wants of the two wants for potentials in the LPA' (including a dependence in the anomalous dimensions) After rescaling to dimensionless variables, the flow equation for the two potentials are given **by** *viêdewwd* d^2 $2 + n$ ² *^v ^h*˙ = (⌘ 1) *^h* ⁺ *d* 2 + ⌘ ² *^h*⁰ ⁺ **C ENTRE PRODUCTION CONTRACTOR** 42*h* ¥ *h*0 iti
10 *d*+1 (1) 4 h *h 6* h *h* **100** where we have defined the constant *C^d* such that *C*¹ giving the scaling solutions in the coupled system of the coupled sy ordinary di↵erential equations 0 = *dv* + d 2 $Z_{\phi,k}$ the ψ ∂_μ ∂^μ ess V_k^T the following $Z_{\psi,k}$ ∂^μ ψ and ∂_μ ∂^μ is the following: **he day as a construction A** mathematic **The assumed. Steery: 24 at special symmetry: Formulae is a special symmetry on the consideration in the construction of the constant** zy
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The flow the flow the flow the flow the flow the flow the flow equation for the flow equation for the flow the \mathfrak{B} vi ution (750 °C) *d* 2 + ⌘ ² *^v* 0 + 1 0 C d **1 gyfeldau** *d*+2 1 + *^v*⁰⁰ *^X^f* ¹ ⌘ *d*+1 10 0 0 10 1 **(1999)** $\frac{63}{65}$ Z d^4x $\frac{1}{\sqrt{2}}$ 2 ★\ . t_k hat the Wess-Zumino model in the Windows at t_k (2) ϕ when t_k and ϕ ret tyhe UN Variance sover !!! . For systems linked to standard Strategy and the international system which are parameterized by a which are parametrized by a which are one required to the parameterized by $\frac{H}{2}$ the $\frac{H}{2}$ then $\frac{H}{2}$ then $\frac{H}{2}$ then $\frac{H}{2}$ then $\frac{H}{2}$ then $\frac{H}{2}$ then $\frac{H}{2$ spinor transform as ! *ⁱ* and ¯ ! *ⁱ* ¯. A generalization of local interactions with such a stock metry of the symmetry then requires and a representation of the area features in a representation of the dimension *d*. (*N^f , d*) and we define *X^f* = *N^f d* for simplicity. There is also the possibility to d_{γ} under, the transformation, which would require the transformation, which we have a local would require In this work we shall consider the case of an odd Yukawa potential *H*. 1 ZNRK & RUNC ROLL AND THE LARGED FOR THE RUNCH AND THE RUNCH STATE OF THE RUNCH STATE OF THE RUNCH STATE OF THE RUNCH STATE OF THE RUN CONTRACT OF THE RUNCH STATE OF THE RUNCH STATE OF THE RUNCH STATE OF THE RUNCH STAT \mathcal{L}_{to} we more in general seen surpresences the form from \mathcal{L}_{to} potentials in the LPA of the LPA's and the anomalous dimensions in the anomalous dimensions of the anomalous di After residual to dimensionless variables, the flow equation for the flow equation for the flow to position for $\frac{v}{v}$ - *dv* + *d* 2 + ⌘ potentials in the LPA' (including a dependence in the anomalous dimensions)) $\frac{\partial \varphi}{\partial n}$ *CHROUTH d*+2 ere, wetterveSAStin ¹ ⌘ *d*+1 1 + *h*² ! **(I.2)** The lagrangian density of the lagrangian density of the lagrangian of the lagrangian of the lagrangian of the and a polynomial form of the scalar polynomial form of the scalar polynomial is used. in O(N)-models at finally reduced was following the following temperature in the finite temperature in Directio
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We go the typical settlement of the typical settlement given by a lagrangian settlement of the two property of **Lettre** 1 ² @*µ*@*µ* ⁺ *^V* () + ¯ *^µi*@*^µ* ⁺ *i y* ¯ (I.1) **M Que in the and** r
E **d4** ÿØ 7 2 $\frac{1}{2}$ $\$ First symmetry: *U*(*N^f*) is assumed. There is a special symmetry one may consider which is the **Z2 system requirements to standard the invariance of an accuracy requirement of the invariance of the concentration of the international system of the international systems of the international systems in the internationa** system which are parametrized by a parametrized by the parametrized by the parametrized by $\frac{1}{2}$ then $\frac{1}{2}$ t spinor transformation as I and ^a interaction as interaction of local interactions with such as Chronology inter
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HARCHIPOW CLK FWC 1944 HOURHELL CONSIDER THE CASE OF ALL PLATE THE FIRE WE OPEN PERSECT WAS SHALL MAKE OUR COURT OF THE DISTRICT OF THE DERIVATION (LOWEST OF THE DESCRIPTION OF THE DEPARTMENT OF THE For a SUSY model both the scalar potential and Yukawa interactions in the only are dependent on the same function. The same function of polynomial truncation in the same function of polynomial truncations has been done. $\rm g$ a $\rm g$ κ φ φ μ φ is κ φ is φ if φ is φ if κ (φ) φ is φ if φ is φ is φ if φ is φ i to a gas of the sign bossis is a show we show with we show it Sect. The Stefan and it of the Stefan Boltzmann Law in the Stefan Ann Law in the Stefan An three dimensions respectively the constants the fermion do not contribute to the flow equation of the flow equations since the since the state to the state that they have a thermal zee chacasions to serve the we there we the dimensional reduction of the bosonic part of the model due to edule to the presence of a thermal series of a thermal zero-mode. We show Sect. The Principal sich algebraic to n. 19, the Ocsomic Portin In a way main a similar way displaced the similar reduction of the simulation we compute the phase diagram for the restoration of the restoration global Z² symmetry at finite temperature in Sect. IV C. IN THE NEW YORK OF THE NEW YORK AND THE THE TWO MODEL There are many simulations of the NST works of the supersymmetric West Zumino models in both four and two space-time dimensions. STOD Two-dimensions. Actually the two-dimensions of two-dimensions of two-N = 2 styles in organisation of the top the top the top the top the top the top to cation of the four-dimensional $\mathcal{N} \rightarrow 1$ model. The $\mathcal{F}_{\rm n}$ dimensional model with N = 1 supersymmetry, on the 1 supersymmetry, on the 1 supersymmetry, on the 1 supersymm useful to result the construction of the three-dimensional model starting from the real superfield $\frac{1}{4}$ **απομ**ογραφία του στ with real (pseudo) scalair fields tegnalism IVI for and dipinor 1 2 oddfyndar 20 oner 5Ka4dp.
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↑ , kT& f& fax the typical settlement and the typical settlement was a lagrange to the type to the type **LEE** \$ ² @*µ*@*µ* ⁺ *^V* () + ¯ *^µi*@*^µ* ⁺ *i y* ¯ (I.1) The truncation we consider is the following the following *k* ⇥ *, ,* ¯ ⇤ = ZZ
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17 *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* ¯ *^µi*@*^µ* + *i Hk*() ¯ ◆ **TRID** There is a special symmetry of the a special symmetry of the *Z2 symmetry required* the *Z2 symmetry requirement* invariance over ! . For systems linked to standard Yukawa system which are parametrized by *H*() = *y* which are odd under *Z*² then one requires that spinor transform as ! *i* and After rescaling to dimensionless variables, the flow equation for the two potentials are given In this work we shall consider the case of an odd Yukawa potential *H*. $\frac{1}{2}$ is . i.i. \pm 14 e hp D alization of local interaction of local interaction of local interactions with such a symmetry than $\frac{1}{2}$ *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define *Xf* α of α for simplicity. The possibility to have the possibility of α in α is a line is a lin transformation, which would require an even function of the production of the west we shall consider the ward of case bin g_{D. 2}1. New York Property and H. (Not sure we want to want to wake the flow in activities for your and want the flow equation for the flow ∂y <u>une d ∂y </u> 1). A 2 S VIIIIII CU jit er Steck ali 1935 945 projektaja je SC very hinteral Yukawa interaction i suktions vero
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Dirac fermions of the scalar field and property of the state fermions of a state of a *k* $\frac{1}{2}$ $\boldsymbol{\psi}$ Z (\mathbf{m}) There is a special symmetry one may consider which is the *Z*² symmetry requinring the invariance over !! . For systems linked to standard you show the system which are parameterized to standard y **by Y / 200 P / 10 M M M N T 200 C 200 M T 200 M T B W D D T T 1 Y 45 (P H 8 S THE T 200 T 200 AS " 12 T 200 AND Green Contraction Mateur Products to 20 North February Production** Steep - 112-4 39 N11110 UPICS , 39 F+F
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一 **Z2 symmetry requirements of the invariance of the invariance of the invariance of the invariance of the invariance
The invariance of the invariance o** system which are parametrized by *H*() = *y* which are odd under *Z*² then one requires that spierro transform as interaction as it leads to the procedure of a general interaction of a general interaction
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The dintensional editional the invariance invariance in the invariance of the invariance of the invariance of system which are parameterized by a which are parametrized by the parameterized by a which is necessary and the spinor transform as ! *ⁱ* and ¯ ! *ⁱ* ¯. A generalization of local interactions with such y a the two dromings Yaln of Dirac fermions in a representation of the scalar potential form of the state in a polynomial form of d_{γ} un d_{γ} , d_{λ}) d_{λ} and sychological d_{λ} , which is also the possibility of the possibility of the possibility of d_{λ} have unched spitted to the transformation, which would require an even function of the transformation of the trans I make our ears in the LPA and make the Little of the derivation of the derivative expansion of the derivative $Z_{\rm{to}} = Z_{\rm{in}} - 1. Z_{\rm{in}}$ and itin exercise $\chi_{\rm{in}}$ anomal corresponsions in $Z_{\rm{in}}$ re we want to write more more in general to write more in general. p_1 the LPA' (including a dependence in the anomalous dimensions) in the anomalous dimensions of \mathcal{W} $d\overline{v}$ **example VERFREEDE ECOLITY** ¹ ⌘ *d*+2 11009444620727 Sapara **detail** 1 + *h*² 1 **(I.2)** The lagrangian dech second the lagrangian density of the lagrangian dechanging the construction of the lagrangian of the lagrangian dechanging the construction of the lagrangian of the lagrangian dechanging the constructio Gills, Petr 1910 (december 2010 (d=4) , Rosa, Vitale, Wetterich 2001 , Wetter 2011 , Constitution and Constitu **Ighan Greek Acht ar** WE GO COC START THE THE THE TWO BEGINS TO BE THE TWO BY A LAGRANGE THE THE TWO BY A **Latin** 姚州 2 @*percent to a million of the designation* The true state is the trumpy of the truncation we consider it is the following interaction and the following:
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Braun, Wipper 2010 TALE PERIODI** 2 *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* First Library Cyfl (4) is a straighead symmetry of general symptometry on the magazine is a special symmetry o
Function of the plant is the lands of an Joves Mills was stational average which is the a special symmetry of Z2 symmetry required to the invariance of the invariance of the invariance of the invariance of the invariance system which are parameter which are parameterized by the parameterized by the parameterized by $\frac{1}{2}$ spinor transform as interactions with the interaction of the dimension of the such as interactions with α interactions with α in α in a symmetry then requires and a symmetry then requires and a representation of the property of the area of the
The New York Mention of the area fermions in a representation of the property of the property of the property of dimensions and we have a formulation of the possibility of the possibi have unchanged spinors under the transformation, which would require an even function *H*(). In this work weishall consider the case of species of an odd Yukawa \mathcal{F} is supported by \mathcal{F} and \mathcal{F} and \mathcal{F} with \mathcal{F} and \mathcal{F} and are dependent on the same function. A study in the same from the same function of local truncations with such θ w. flow the fact of the procedure resides fact that the finite views equ uhei \mathbb{Z}_2 ssyformetaionphasinb equ a equ we to be equ the of the position of the complete of the Contribution of the to per gas had a gas of matter to conserve the states of the states Section Hotel Hotel Stefan - In Koontendent Law in Board in Content of August 1979 three dimensions through the fermions the fermions the fermions of do not contribute to the flow equations since the flow equations since the flow equations since the since the have a thermal zero-mode. On the other hand we observe the original control on the other hand we observe the o dimensional reduction in the bosonic part of the model due to the presence of a thermal zero-mode. We show in the presence of a thermal α thermal α Side of write do always that the AW of the CHVG always weaks In a similar way dimensional reduction has been observed by the similar way of the similar production of the similar states of the sim $i\in \mathcal{O}(d)$ in $i\in \mathcal{O}(d)$, $i\in \mathcal{O}(d)$ is a finite temperature in $i\in \mathcal{O}(d)$. Finally, $i\in \mathcal{O}(d)$ is a finite temperature in $i\in \mathcal{O}(d)$ is a finite temperature in $i\in \mathcal{O}(d)$ is a finite temperature in WE COMPUTE THE PHASE DUTE TO THE PHASE DIAGRAPHY TO THE PROPERTY THE RESTORATION OF THE RESTORATION OF THE RES pal Capitan Capital Temperature in Sect. In Sec IN THE N EXPLORATION CONTINUES THREE DIMENSIONS AT THE PROPERTY OF THE UNITED PACTER ARE MANY WORKS ON THE SUPERSYMMETRIC WARDED TO THE SUPERSYMMETRIC WAS COMP **Zumino models in both four and two space-time di**mensions. Actually the two-dimensions. Actually the two-dimensional model with the two-dimensions. N = 2 supersymmetries is the top to the top the top to the top to the top to the top the top to the top to the
Next towns of the top to the top the top the top the top to the to cation of the four-dimensional Transition of the three-dimensional Transition of the terms of the three-S ASPOCI POSOVOLIST SO "CMITETIST") LIESS MARSIOLITORI IN CRIMINING MARINAMENTO MENTO MARINAMENTE PSYRAPROMA M
1) H12ONY TRE FONT HIMLER GRAFIST PRIMERING EN HANGER REGALISMENTO SOLOMO IN TERRA HIMLER SENTIMENTE POUR REGER of the correct dictive of organism and the computations of the computations of the computations. use full the computation of the construction of the three-dimensional model starting franches from the ready Φράλου - φράλου - φράλου - φράλου - φράλου 2 αριστιά στο Συγγραφία
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A the Ø_DGV pyptane wikighte standave *sifawbe*r 2α dk+ 2α or 2α which α and **s** of the state a constant to the constant of the state of the state will make. That in obeys the settlem addition and the settlem and the settlem and the settlem and the settlem such ភ្
ឆ $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$ The water of mathematic and interaction term is the D term of 2W of 2W $\frac{1}{2}$ $\frac{1}{2}$. and $\frac{1}{2}$ is a finite strain of $\frac{1}{2}$ and $\frac{1}{$ **SLIN + Hattage** $\frac{1}{2}$ $\frac{1}{2}$ $W_{\rm q}^{\prime\prime}$ The complete of the complete off-shell and takes the simulation of the simulation
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W**\6@r**
W\1\aN From this expression we read of the wind that we have a self-interaction potential for the scalar fields. For a self-interaction potential for a self-interaction of polynomial superpotential which the power of the leading term is even, ^W(φ) = ^cφ²ⁿ ⁺ ^O(φ²ⁿ), we do *k* ⇥ *, ,* ¯ ⇤ = not observe supersymmetry breaking in our present nonporture renormalization group study and cooking hand spontaneous supersymmetry breaking is definitely possible for a superpotential in which the possible for the possible point of the power of the possible point of the possible contract of the possible contract of the possible contract of the possible contract of the possi leading term is one of the explicit calculation of the explicit calculations we shall be explicit calculations
The explicit calculations we shall be explicit calculations we shall be explicit calculations were also assume use a Majorana representation for the verment and the verment **σ1313/4016134 http://www.citter.org/** II. FLOW EN THE STATE OF THE TELL AT T Here we shall study the following the following the following the following the following the model V is λ is Ω is λ is λ is λ is λ is λ is λ interactions (not considerably and considerations in the consideration of the constant of the constant of the c
100 which may be a straight from the constant of the constant of the constant of the constant of the constant I. FORMAL ANDERS FORMA We go beyond the testor ation by a lagrangiant the typical settlement of the test of the test of a lagrangiant g 4 ² @*µ*@*µ* ⁺ *^V* () + ¯ *^µi*@*^µ* ⁺ *i y* ¯ (I.1) The truncation we consider the following the following of the following of the following of the following of the following: The following of the following Д∰
Эг d4*x* DE 2 **教授** *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* ¯ *^µi*@*^µ* + *i Hk*() ¯ **Z2 system or de for system and the invariance of the invariance of the invariance of the invariance of the inva
Zapire virked to standard Zeder by Fize (***O)* **in Existema wantampartes odded to the invariance of the invaria IT OTH THE TRIZET BUDELICH CHENDELISHOH TAG ARE PART FILM MATHEMSIGH TO DHA PLANE PART BY WHICH ARE ONE REQUIRED TO PART THE PART OF PART O** spinalized transformation as it is the state of a great transformation and the spinalization of the state of h
The original proposition is the sympth of the rest of Local interactions with the rest wall in the local is the a symmetry then requires an odd *H*). There are *N^f* Dirac fermions in a representation of **IQ FORMULAE** The truncation we consider the following: ← UELY DU RENDE \mathbf{f} d4*x* 江
1521 **25** *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* ¯ *^µi*@*^µ* + *i Hk*() ¯ や (I.1) There general symmetry of the *industry one may consider the symmetry requirements* and the *Z2 symmetry requirement* invariance over 23 . For systems linked to standard your system which are parameter to standard you which are p by *Hart are one for a green for the discussions* and the position of the spinor transformation of the spinor transformation and the spinor transformation of the spinor transformation of the spinor transformation as in the and interactional interaction of dimensional control instruction of the property of the symmetry of the and odd
Pelo Rough delicities in a symmetry then republican requires and the process and duck Nukwassal oddieut, And *H*). There are *N^f* Dirac fermions in a representation of dimension *d*. (*N^f , d*) and we define **Xf** α for simplicity. The simplicity is a form of the possibility of the possibility of α and $\$ transformation, which we shall consider the second with the consideration of the state of the consideration of sion, with *Z and the Z and the complete the complete stream and the complete and the complete stream and the complete and the complete stream and the complete stream and the complete stream and the complete stream and the* (Not sure we want to call the flow with the flow as a michigan valid flow equation for the two streams in the two potentials in the LPA' (including a dependence in the anomalous dimensions) in the anomalous dimensions in the After descaling to dimension and flow scould be made to dimensionless variables, the two potentials are the two **R 48 DEX LOUGE FOR LOCAL FOR LEAS NICE RECENT WORKER MESONIC IT ESTAVELS CONSIDERATIONS INTERACTIONS MORE GENE** awle Rade below we all the thing We study the model at critical terms of the model and the flow of the effective average acceptive in a local potential approximation (LPA) and the following the following the following the following the following the following truncation (LPA)
and the following truncations in the following truncation (LPA) + (eventuall α depends to the dependent of α of the dot α of α and α in α for α and α for extension with α to the α ature **tiquipe of the log strume way provided betaves**, subtailly the toble flow edge functions for the persidicit the district core was wanted the character of the the set of the struck frame as v^{2-2} was and the \sim her many of thus performance and the state of the support of the support of the state and the state and the state of o from the theory space. The theory space is the theory space of the systematic space. The theory space is the strategies in the strategies and appropriate in the strategies in the strategies and appropriate in the strateg dine in the control recurring in the oceanic **port of the vertex expansion** of compare or the derivate will allow **RACACALLES** $\frac{1}{2}$ d*dx* ₹ 稱 *Z,k* @*µ*@*µ* + *Vk*() + *Z ,k* ¯ *^µi*@*^µ* + *i Hk*() ¯ STP
「1 **. (II.2) 217 Ballis** UH SHER EN UHT HERE SUPERTHERE STATISTICS TO SHER HERE IS A REAL COMPLICE THE REAL CONSTRUCTION OF A REAL COMPONENTS. The latter parameter is relative to the system of the system and plays the τ and the simulation pure fermion of the index of the state theorem in the state of τ in τ as we truncate the second of the second of the state the state truncate the state truncate τ as τ as τ as τ a the space to the ansatz of Eq. (II.2), for the mechanism of Eq. (II.2), for the mechanism of α and α and α in α is α and α is α is α is α in α is α is α is α is α is α is α i we can simply deal with the total with the total can simply dependent of the total deal $\frac{1}{2}$, $\$ conservation and this assembly real number of the truncation as a structure in the truncation above is missing a
The truncation above is missing to the four-dimensional production as a line is missingly real number is moth purely fermionic derivative-free interactions, that are indeed symmetry-sensitive and that would *^h*˙ ⁼ *^h* (⌘ 1) + *^d* 2 + ⌘ ² *^h*⁰ *^h*˙ ⁼ *^h* (⌘ 1) + *^d* 2 + ⌘ ² *^h*⁰ + 2*v^d* **h** Truncation: Physics also for interacting fermion systems, for SUSW models are the fluod of the relations of archite is equipmented and material the second of the line of the linear regulator and the line in the linear regulator of the line of the line simple in e annihiperemputation of the form of the flow ϵ and r the flow equation of the flow equations of the two equa rential wapt at New York more in the **deditional** ² *^v* ⁰ + *C^d* ¹ ⌘ *d*+2 1 + *^v*⁰⁰ *^X^f* ¹ ⌘ *d*+1 1 + *h*ave θ . **TIME 1** $+\partial_d^d$ 2*h* ل
ع \mathbf{f} **h** \mathbb{R}^2 ¹ ⌘ *d*+1 (1 + *h*2) ² (1 + *v*00) $\frac{46}{11}$ **1 e 16 Fre**
1 STANA *d*+2 **(1 +** *h* **+** *b* + *x* + *y*⁰) (*h* + *x* + *x*⁰) (*h* 2 **24** re
Pr **h00** ¹ ⌘ **d**
1988
Adalla (*11ews)@hy* 9 5 **SALE REGIONAL** Symmetries: v even and *denoted for convenience* for problem: (*a, X*^{*n*}) and the problem: (*d, X*^{*j*}) and d) and d *d, X<i>f*) and d (*d, Xx*)) and d (*d, Xx*)) and d (*d, Xx*)) and d (*d, X* With real (pse or fairly punctic the stability of the stability of the vacuum is the requirement of \mathcal{E}_1 \mathcal{P} of \mathcal{P} and \mathcal{P} and \mathcal{P} and \mathcal{P} and \mathcal{P} and \mathcal{P} \mathcal{P} \mathcal{P} \mathcal{P} and $\lim_{\epsilon\to 0} \frac{d}{d\alpha}$ whenever the wave $\lim_{\epsilon\to 0} \frac{d}{d\alpha}$ and $\lim_{\epsilon\to 0} \frac{d}{d\alpha}$ and $\lim_{\epsilon\to 0} \frac{d}{d\alpha}$ and $\lim_{\epsilon\to 0} \frac{d}{d\alpha}$ the fugure the fields have not the finite of the fields of the field of the case of the concentration of the inclusions of the inclusion of the inclusion of the inclusions of the inclusion of the inclusion of the inclusion h atter will be named Latter with h at the choice of the contribution for the choice of the exhaustine of the choice of the choice of the exhaustive evidence that similar and similar and the existence of the existence and properties of the existence and conformal conformal c **dels in 2010 157655 tunear sylling for line as of finite symmetry with styllights w**ith styllights with styling Projection of the Wetterich equation of the Wetterich equation of Truncation of Eq. (II.2) yields the running o the corresponding parameters. Since we are interested in reproducing the corresponding in reproducing the cont correspond to scale the CAM OF the RG flow the RG flows of the RG flow, it is useful to consider the CHI COLL **BUTTER** *desturities Z*1*/*² *,* ! *k*(d)
k(d)**/**210
*d*100122 **Z** \mathbf{h} $\partial \theta$ in fail in T would T extend $\partial^{\mu}\partial \partial^{\mu}\mathcal{D}^{\nu}$ if the constant $\partial^{\mu}\mathcal{D}^{\nu}$ and $\partial^{\mu}\mathcal{D}^{\nu}$ and $\partial^{\mu}\mathcal{D}^{\nu}$ and $\partial^{\mu}\mathcal{D}^{\nu}$ and $\partial^{\nu}\mathcal{D}^{\nu}$ and T is it is consequence we will focus on the potential of the property of the property of the property of the property of *v d*₂ *kx d*₂ *kx d*₂ *kx d*₂ *kx d*₂ **THE TO DRE** *k*(*d*2)
*ion: d*2 | **, heads of its y and a strong property of the and a strong property of the angle of the ang ZZZZ** *H^k* **C 21/12/2025/**
24/20 23/24/20 *k(0)* Q13)
{})} d2 X / $\frac{1}{46}$ *.* This new set of variables the flow equations reading the flow equations reading the flow equations reading the *vine # d* 2 + ⌘ ² *^v* ⁰ + 2*v^d* n
D *l* (B)*d* ⁰ (*v* ⁰⁰) *X^f l* (F)*d* ⁰ (*h*2) o HSI3D **n** 2*h*(*h*0) 2*l* (FB)*d* ¹*,*¹ (*h*2*, v* ⁰⁰) *h*00*l* (B)*d* **1 (***v i* **lut as 00** o **Ciritt** ⌘ ⁼ ⁴*v^d d* **ie** *<i>x*(a)))393
2. **TCCLC** *mous di*
QQDS10 151011 ⁰⁰)+2*X^f* (*h*⁰) I) h **steractic** ⁴ (*h*2) *^h*2*m*(F)*^d* 2 (*h*2) **in** 10 1601 $\frac{1}{2}$ $\frac{1}{2}$ *d* \mathcal{V}^{\prime} (*h*0 \oint_{0} ²*m*(FB)*^d* 11,2 (*h*₂ $\mathbf{d}% _{0}$ **By** 21 **KDRGC** Rescaled to Macdes Maria dimensionles Flow equation for linear optimised regulators Two functions

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potentials in the LPA' (including a dependence in the anomalous dimensions))

The truncation we consider is the following:

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relevant at the corresponding critical dimensions: Numerical analysis

Multicritical structure dictated by the marginal interactions, analysis with canonical dimensions

$$
\phi^{2n} : d_c^v(n \ge 2) = \frac{2n}{n-1} = 4, 3, \frac{8}{3}, \frac{5}{2}, \frac{12}{5} \cdots
$$

$$
{}^1 \bar{\psi}\psi : d_c^h(n \ge 0) = \frac{4(n+1)}{2n+1} = 4, \frac{8}{3}, \frac{12}{5} \cdots
$$

Since we look for odd Yukawa potentials, we can restrict the list of the operators that become

to polynomial truncations of the potentials, see Sect. VI.

 ϕ^{2n+1}

Numerical analysis from the asymptotic region

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A

At large field values one can construct the asymptotic expansion of the solution as a function of free parameters and then evolve numerically towards the origin imposing the known boundary conditions $v'(0) = 0$ $\frac{1}{100}$ $h(0)^{10} = 0$

Some properties of the fully non trivial LPA scaling solutions in d=3: if X_f < 1.64 the scalar is in the broken phase.

4000

6000

8000 B **EXECUS OF the solutions in the range of the solutions in the** 10³ *< X^f <* 3. One can notice the transition from the broken to the symmetric regime, which occurs *n* plane (σ, h_1) as function of X_f

Vacuum

(0)) from the numerical global scaling solutions, varying *X^f* in the range

(1 + *^y*)² *^y* ¹

10³ *< X^f <* 3. One can notice the transition from the broken to the symmetric regime, which occurs

 $h'(\phi_0)$

(*d* 2) *v*⁰ + *C^d*

Polynomial truncations $\frac{100}{18}$ a consequence, in this regime it is necessary to change also the parameterizations of *h*() and \overline{y} Though, in general, is no special point for the function *h*(), it still enters in the definition of $\frac{1}{2}$ the vertex functions, from which one extracts the physical multi-meson $\rho = \frac{\lambda^2}{2}$. $\frac{1}{2}$ **d** $\frac{1}{2}$ 2*.*377 10² 1*.*793 10² 1*.*253 10² 7*.*316 10³ 2*.*171 10³ 1*.*164 10⁴

the potential and the Yukawa function, in the LPA⁰

exchange interactions (right and left panel respectively).

regards of which is in the symmetric induction of $\mathcal{L}_\mathcal{A}$ function of $\mathcal{L}_\mathcal{A}$ function of $\mathcal{L}_\mathcal{A}$

regime, the physically meaningful parameterization of the scalar potential is a Taylor expansion

n=1

$$
\rho = \phi^2/2 \qquad y(\rho) = h^2(\phi)
$$
\n
$$
v(\rho) = \sum_{n=0}^{N_v} \frac{\lambda_n}{n!} \rho^n
$$
\n
$$
v(\rho) = \lambda_0 + \sum_{n=2}^{N_v} \frac{\lambda_n}{n!} (\rho - \kappa)^n
$$
\n
$$
h(\phi) = \phi \sum_{n=0}^{N_h - 1} \frac{h_n}{n!} \rho^n
$$
\n
$$
h(\phi) = \phi \sum_{n=0}^{N_h - 1} \frac{h_n}{n!} (\rho - \kappa)^n
$$
\n
$$
y(\rho) = \sum_{n=1}^{N_h} \frac{y_n}{n!} [(\rho - \kappa)^n - (-\kappa)^n]
$$

y → → X(→ X(→ X → X)

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⇢*ⁿ .* (VI.3)

the vertex functions, from which one extracts the physical multi-meson Yukawa couplings. As

ⁿ . (VI.4)

, with order \mathcal{M} , with order meson-

we found that the two strategies nicely agree, so that (*N^v* = *D, N^h* = *D* 1) is a very good

a consequence, in this regime it is necessary to change also the parameterizations of *h*() and The pair (*Nv, Nh*), or more generally an ordering of the polynomial couplings by priority of inclusion in the truncations, can be chosen by relations, can be chosen by relations, as in an analysis of $\mathcal{L}_\mathcal{S}$ Expansions: around the origin or non trivial vacuum (I,II) vs numerical ODE sol. d the origin or non trivial vacuum (I,II) α α α β *n*=1 *n*! [(⇢) *ⁿ* () *ⁿ*] *.* (VI.6) s. around the orig

^v(⇢) = ⁰ ⁺^X

*n*2

n!

TABLE IX: Case *d* = 3 and various *X^f* , polynomial expansion of *y*(⇢) around the non-trivial (left panel)

corresponding perturbations. Alternatively, and maybe less economic scan over the $\frac{1}{2}$ σ to $Z_{\iota}(\phi)$ and $Z_{\iota}(\phi)$ not so useful, *Knori* Phys. Rev. B94(2016) 245102 ded 4 derivative expansion for momentum dependent vertex expansion. $\frac{1}{2}$ who consider the could include the latter and consider the pairs (*N*^{*y*} $\frac{1}{2}$, $\frac{$ \overline{X} **h**
h(*h*) strong impro Strong improvement comparing to results obtained with a smaller truncation with fixed $h(\phi) = h_1 \phi$ *y* probable *n*_b *x*
*ab*lv *y* bably needed 4 derivative expansion for momentum dependent vertex expansion. Moving to $Z_{\phi}(\phi)$ and $Z_{\psi}(\phi)$ not so useful, *knorm* Phys. Rev. B94 (2016) 245102 probably needed 4 derivative expansion or momentum dependent vertex expansion.

we found that the two strategies nicely agree, so that (*N^v* = *D, N^h* = *D* 1) is a very good

31

we would expect the pairs of pairs (*NV* λ), i.e. μ and the pairs (*N* μ), i.e. μ), i.e. μ), i.e. μ (i.e. μ), i.e. μ (i.e. μ), i.e. μ

Effective average Hamiltonian action \mathcal{A}_{\bullet} Effective average Hamiltonian action effective Hamiltonian action, which is a trivial generalization of the more widely known \mathcal{L} ! $\frac{1}{2}$ α

!

field configuration in (26), using the definition (27) and integrating out the momenta, one

One can study the quantum/statistical field theory in phase space. bosonic degree of freedom government beautocreat Trese μ and the fold the same in the same canonical control μ and μ and μ and μ and μ $\frac{1}{2}$ statistical field theory in phase-space. The following path integral: effective Canactic Lagrangian action. The latter Canactic intervals in the configuration of the configurationspace path integral external sources J coupled to the Lagrangian variables, and by taking can study the quantum/statistical field theory in phase space. G.P.V., Zambelli Phys. Rev. D86 (2012) 085041 such that we need Rk → 0 and μ and μ when k μ as well as μ when k μ μ μ μ μ

 $\mathcal{L}^{(2)} = \int [apaq] \mu[p, q]$

$$
S[p,q] = \int dt \left[p(t)\partial_t q(t) - H(p(t), q(t)) \right]
$$
\n
$$
e^{\frac{i}{\hbar}W[I,J]} = \int [dpdq] \mu[p,q] e^{\frac{i}{\hbar}\{S[p,q]+I\cdot p+J\cdot q\}}
$$
\n
$$
\Gamma^H[\bar{p},\bar{q}] = \text{ext}(W[I,J] - I\cdot \bar{p} - J\cdot \bar{q})
$$
\n
$$
e^{\frac{i}{\hbar}\Gamma^H[\bar{p},\bar{q}]} = \int [dpdq] \mu[p,q] e^{\frac{i}{\hbar}\{S[p,q]- (q-\bar{q})\cdot \frac{\delta\Gamma^H}{\delta\bar{q}} - (p-\bar{p})\cdot \frac{\delta\Gamma^H}{\delta\bar{p}}\}}
$$

therefore from the usual quadratic form

²π! .Also one can easily extend all the

Itive tecnn s are easily extended Perturbative techniques are easily extended. 2 **Example 18 as a mathematical parameter effective techniques are easily extended.**

 \mathbb{R} as k \mathbb{R} as traditional, to keep the framework as simple as \mathbb{R}

out of an odd differential operator or diagonal and built out of even differential operators.

2 The effective Hamiltonian action in quantum mechanics

obtains

formalism to an euclidean description. Since we want to keep our discussion as general as possible we will not specify the precise space of functions on which the functional integral

 I,J

be quadratic in the fields

"

Det ^R^k

#

nalization group for the action written in terms of the Hamiltonian. Define an effective Hamiltonian flow. The coarse-graining is in the full phase space. uon grot Perturbative techniques are easily extended.
Wilsonian renormalization group for the action written in terms of the Hamiltonian. Define an effective Hamiltonian flow The coarse-or σ are seen are several ways to convince ourselves that for the functional one can be σ Define an effective Hamiltonian flow. The coarse-graining is in the full phase space. equivalent to the Hamiltonian Dyson-Schwinger equations. In fact, the identities: Wilsonian renormalization group for the action written in terms of the Hamiltonian. $\frac{1}{2}$ $\frac{1}{2}$ Define an effective Hamiltonian flow. The coarse-graining is in the full phase space. of the Hemiltonian ′ ll phase space.

$$
e^{iW_k[I,J]} = \int [dpdq] \mu_k[p,q] e^{i\{S[p,q]+\Delta S_k[p,q]+I\cdot p+J\cdot q\}} \qquad \Delta S_k[p,q] = \frac{1}{2}(p,q) \cdot R_k \cdot (p,q)^T
$$

$$
e^{i\Gamma_k[\bar{p},\bar{q}]}=\int\left[dpdq\right]\mu_k[p,q]e^{i\left\{S[p,q]+\Delta S_k[p-\bar{p},q-\bar{q}]-(p-\bar{p})\frac{\delta\Gamma_k}{\delta\bar{p}}-(q-\bar{q})\frac{\delta\Gamma_k}{\delta\bar{q}}\right\}}
$$

$$
\boxed{ i \dot{\Gamma}_k = \frac{\dot{\mu}_k}{\mu_k} + i \langle \Delta S_k [p - \bar{p}, q - \bar{q}] \rangle_k} \qquad \qquad \Gamma[\bar{p}, \bar{q}] = \int \! dt \, (\bar{p} \partial_t \bar{q} - H_k [\bar{p}, \bar{q}])
$$

 $\delta \hat{q}$ $=$ $\delta \hat{q}$ $=$ $\delta \hat{q}$ $=$ $\delta \hat{q}$

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 $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$. $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$, $\mathcal{D}_\mathcal{A}$,

gests the appropriate k-dependent deformation of the function of the function of the functional measure: if the

functional [14]. Following this line of thought we can guess a convenient choice for the

[∂]p∂^q are constant (the latter is

For zero sources one has the equations for the equations for the vacuum configuration (η

δΓk "

and ask for ^µ^k exp{i∆Sk} to become ^µ as ^k [→] 0 and to provide a rising delta functional \mathcal{A} s k \mathcal{A} as \mathcal{A} traditional, to keep the framework as \mathcal{A} to \mathcal{A} to \mathcal{A}

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as a mathematical parameter unrelated to a physical sounding coarse-graining procedure,

out of an odd differential operator or diagonal and built out of even differential operators.

symplectic potential is λ^k = p(1 + rk)dq, the new non-trivial Liouville measure would

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trant

 2×2 , where α is the regularized symplectic form. This choice is the regularized symplectic form. This choice is the regularized symplectic form.

. (6)

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ϵ Lagrangian action. The latter ϵ is defined by introducing in the configuration- $\mathbf{n} \left(q,p \right)$ $\left(\alpha \right)$ $s_{\rm max}$, and $s_{\rm max}$ when $k_{\rm max}$ when $k_{\rm max}$ when $k_{\rm max}$ det af det a \mathbf{v} the proof can be found in \mathcal{L}_p . The found in \mathcal{L}_p **b** Example of regulators in (q, p)

pārtas

=

Rk (transferred)

 $T_{\rm eff}$ respectively respectively reading α

 \mathbb{R} , \mathbb{R} , \mathbb{R} , \mathbb{R} , \mathbb{R}

δJt′

δp¯t

 \mathbb{R} . The set of \mathbb{R}

=

 $\tilde{}$

^δp¯ ^δ^I

δIt′

The first choice can be interpreted as a k-dependent deformation of the symplectic po-dependent of the symplectic po- $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{2}$

Example of regulators in
$$
(q, p)
$$

\n
$$
R_k(t,t') = \begin{pmatrix} 0 & r_k(-\partial_t^2)\partial_t\delta(t-t') \\ -r_k(-\partial_t^2)\partial_t\delta(t-t') & 0 \end{pmatrix} \qquad \mu_k = \begin{bmatrix} \text{Det} \frac{1}{2\pi} \begin{pmatrix} 0 & (1+r_k(-\partial_t^2)) \partial_t\delta(t-t') \\ -(1+r_k(-\partial_t^2)) \partial_t\delta(t-t') & 0 \end{pmatrix} \end{bmatrix}^{\frac{1}{2}}
$$
\n
$$
R_k(t,t') = \begin{pmatrix} \mathcal{R}_k^p(-\partial_t^2)\delta(t-t') & 0 \\ 0 & \mathcal{R}_k^q(-\partial_t^2)\delta(t-t') \end{pmatrix} \qquad \mu_k = \begin{bmatrix} \text{Det} \frac{1}{2\pi} \begin{pmatrix} \mathcal{R}_k^p(-\partial_t^2)\delta(t-t') & \partial_t\delta(t-t') \\ -\partial_t\delta(t-t') & \mathcal{R}_k^q(-\partial_t^2)\delta(t-t') \end{pmatrix} \end{bmatrix}^{\frac{1}{2}}.
$$

and calling α the extremal point, it is straightforward to show that α

²)∂tδ(t − t′

[∂]p² and [∂]2^H

 \equiv δ2Γ˜^k

Det ^σ^k

1

= −

+

Γ˜(2)

I = DIE OOK

= −

A different choice which makes the computation of the computation of the traces even simpler than for a simple \Box Flow equations \Box $t \cdot t \cdot \infty$ is a sequence $t \cdot \infty$ T_{OM} equations T_{OM} and (36) finding that singularities appear at nonvanishing values of k. This happens because at some k the radius of convergence of the necessary expansion of the r.h.s. in powers to zero, a fact relations to \sim from the terms \sim for the terms \sim for the fields in the fields i the bare Hamiltonian of the n = 2 model. If no expansion is performed, as in the numerical Here one could adopt any of the regulators R^k developed in the vast literature about the $F|_{0}$ Flow equations

 \mathbf{r} Local Hamiltonian approximation (constant \bar{q} , \bar{p}) because at some k the radius of the radius of the radius of the necessary expansion of the r.h.s. in the r.h.s **integration** integration of the terms of the terms quadratic in the flow equation for \bar{q} , \bar{p}) and the flow equation for \bar{q} , \bar{p}) $\frac{1}{2}$ can be estimated by the value of $\frac{1}{2}$ I_{α} and Hamiltonian approximation (constant \bar{q} , \bar{p}) only in quantum mechanics, because it does not produce any coarse graining any coarse graining and therefore graining any coarse graining and therefore graining and therefore graining any coarse graining and therefore gra \bar{q} , \bar{p}

 $\mathcal{P}(\mathcal{P}^{\mathcal{P}})$ in $\mathcal{P}^{\mathcal{P}}$ (p2)/2 of finite order. Indeed if the bare Hamiltonian depends on $\mathcal{P}^{\mathcal{P}}$

 k (k \geq

 $\mathcal{P}(\mathcal{P}_1, \mathcal{P}_2)$

δ det Historical Community (1999), **1999**

 σ and σ and σ on \mathcal{H} approximation also HHA approximation also HK can be shown to respect to \mathcal{H}

the bare Hamiltonian of the n \mathbb{R} model. If no expansion is performed, as in the numerical \mathbb{R}

polynomial expansion of the flow equation of the flow equation. Such a choice is that of a diagonal regulator,

 \overline{OC} integration for $1/\overline{ID}$ in the flow equation for $1/\overline{ID}$ **Can be estimated by Canadian Can** commute with each other, assuming that the traces can be written as α

2

Tr #

this symmetry, for suitable cutoff operators.

^k(¯p, ^q¯) = ¹

die eerste verklank

transform), and that there is no UV cutoff in the theory, then by Wick rotating the trace

to say an operator which is multiplicative in both time and frequency representations; in

it does not regularize the functional traces. Assuming in the functional traces. Assuming the function \mathcal{C}

 \mathcal{A} different choice which makes the computation of the computation of the traces even simpler than for a

In the LHA and if the second derivatives of H^k commute with each other, this gives the

whenever both Tk and Vk are polynomials of degree higher than two, the degree higher than two, the determinant

becomes a function of both α and α and α and α and α dependence also mixed α

in the effective Hamiltonian. Therefore one should consider a larger truncation in order to

 τ such terms. Also a structure of a structure of a τ model kind, τ and τ

a dependence in the momenta which is more than quadratic. We stress that in general

 ω_{qq} and ω_{qq} and ω_{qq} ω_{qp} ω_{qp} ω_{qp} ω_{qp} $\det H^{(2)}_k = \partial^2_{\bar{q}\bar{q}} H_k \, \partial^2_{\bar{p}\bar{p}} H_k - (\partial^2_{\bar{q}\bar{p}} H_k)^2$ Notice that the second trace vanishes whenever it is possible to evaluate it in Fourier space $\frac{d}{dt}$ these estimates do not reach a great accuracy either because of spurious dependence on $\mathbf{r}(\mathbf{z})$ or $\mathbf{r}(\mathbf{z})$ or $\mathbf{r}(\mathbf{z})$ or $\mathbf{r}(\mathbf{z})$ the boundary conditions (which can be controlled by some non-linear redefinition of $\lim_{k \to \infty} \epsilon = o_{\bar{q}\bar{q}} \Pi_k o_{\bar{p}\bar{p}} \Pi_k - (o_{\bar{q}\bar{p}} \Pi)$

. (35) \sim (35) \sim (35) \sim (35) \sim (35) \sim (35) \sim

k (op, door) = − Tr # r

are 1-by-1 bosonic matrix matrix of the operators in the trace can be simplified and one of the trace can be simplified and one of

 $\frac{1}{2}$

 r

[−]∂2(1 + ^rk)2^δ [−] detH(2)

 \mathbb{R}

to say an operator which is multiplicative in both time and frequency representations; in

other words a function of k and Λ only. If no UV cutoff is possible is possible is possible is possible is pos

only in quantum mechanics, because it does not produce any coarse graining and therefore

A different choice which makes the computation of the traces even simpler than for a

constant regulator regulator is the square root of the Litim regulator $\mathbb{Z}/2$

rk(E2)E = −(k + E)θ(k + E)θ(−E)+(k − E)θ(k − E)θ(E).

+

detH(2)

²)i∂t, and with θ the Heaviside step function, after Wick rotation such

^Pk^δ [−] detH(2)

−∂2(1 + rk)2δ − detH(2)

 s contimized and when the domain in such space is symmetric around the origin. If the original the origin. If the case we can ϵ estent and reach a great accuracy either because of spurious dependence on ϵ t_{total} or because of numerical errors: typically we reached no more than two digit accuracy i the region around the minimum around the minimum spectrum. In order to get stability with a predictions with a problem of α costant optimized

in the effective Hamiltonian. Therefore one should consider a larger truncation in order to

a dependence in the momenta which is more than α stress than α stress than α

2(1 + r)²

 \sim

"

−
−

det Henry r

dt H˙

dr ⁼ [−] ¹

 N s started by started by studying these polynomial truncated flows as generated flows as generated by equations (35) μ

and (36) finding that singularities appear at nonvanishing values of k. This happens

. <u>(34) - Andrea British, angleški koledarju (34) - Andrea British, angleški koledarju (34)</u>

 $\dot{H}_{k} = -\frac{k}{\pi}$ π $\mathrm{det}H_k^{(2)}$ $k^2 + \text{det}H_k^{(2)}$. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972. (36) 1972 $\frac{1}{2}$ dot \overline{d} because of numerical errors: typical errors: typically we reached no more than two digital k de $\frac{d^{2}H}{dr} = -\frac{1}{2(1+r)^2} \left(\det H_r^{(2)} \right)^2$ $H_k = -\frac{1}{\pi} \frac{1}{k^2 + \det H_r^{(2)}}$ α we turn that α different choice of regulators, curing the problem about the problem abo dH_r a det $H_k^{(2)}$ we turn about the problem about the pro as in eq. (31). We chose this regulator to be constant, i.e. κ $-$ + α _e $k = k$ $\frac{dH_r}{dr} = -\frac{1}{2(1+r)^2}$ $\Big(\mathrm{det}H_r^{(2)}$ $\frac{1}{2}$ 2 . The state $H_k =$

 \overline{C} One can study the spectrum of the quantum mechanical models non quadratic in the momenta, which have a non reducible path integral. H^{model} and contained in the moments average effective in the momenta, and plug it is in the momenta, plug it is in the last formula by P ²) . One of the simplest choices for the regulator is a constant rk, that is The can study the spectrum of the quantum mechanical models non One can study the spectrum of the quantum inechanical models hold
which have a non-reducible path integral operator (recall that we are assumed we are assumed with the recognization of the contract of Recall that the recognization of the contract of Recall that the contract of Recall that the contract of Recall that the contrac i study the spectrum of the quantum mechanical models non quadratic in the momen C_{max} and study the succtions of the sympathy mechanical ω One can study the spectrum of the quantum mechanical mo

track such terms. Also a structure of a structure of a structure of a original control in the momenta, generat also introduced a UV cutoff \mathcal{A} in order to control the flow for \mathcal{A} \mathcal{A} Diagonal IR regulator $\frac{1}{2}$ integration of the flow from R \sim $\frac{1}{2}$ $\frac{1$ Diagonal IR regulator

is

 $2 - 7$

 \mathbb{R}

transform of rk(−∂^t

a regulator reads

$$
\partial_{\mathcal{R}}\dot{H}_{\mathcal{R}} = -\frac{1}{\pi}\arctan\left(\frac{\Lambda}{\mathcal{R}}\right) + \frac{2\mathcal{R} + \partial_{\bar{p}\bar{p}}^2H_{\mathcal{R}} + \partial_{\bar{q}\bar{q}}^2H_{\mathcal{R}}}{2\pi\mathcal{D}_{\mathcal{R}}}\arctan\left(\frac{\Lambda}{\mathcal{D}_{\mathcal{R}}}\right)
$$

In this scheme good estimates for the ground state energy E⁰ and the energy gap ∆E¹ = E_1 en by simple polynomial truncations. For a bare Hamiltonian which polynomial truncations. For a \mathbb{R}

$$
\mathcal{D_R} = \sqrt{\mathcal{R}^2 + \mathcal{R} \left(\partial^2_{\bar{p}\bar{p}} H_{\mathcal{R}} + \partial^2_{\bar{q}\bar{q}} H_{\mathcal{R}} \right) + \mathrm{det} H^{(2)}_{\mathcal{R}}}
$$

A quantum mechanical example \forall quantum mechanical ex The function OW in the right hand side of eq. (40) is called the Right hand side of \mathbb{R}

 $A = A$ and a construction of the framework discussed in the previous subsection of the previous subsections of To integrate the flow from the UV to the IR we need to specify the bare Hamiltonian at the UV scale. This is in 1-1 correspondence with Hamiltonian operator, being its Weyl symbol (i.e. Weyl ordered). the fact that the functional integral over the conjugate momenta is not Gaussian. This will To integrate the How-Irom the UV to the IR we need to specify the \mathbb{R}^n . α D we need to consider the bere Hemiltonian et the IW scale. $f(x)$ and the integral of $f(x)$ and the order of $f(x)$ and the order of $f(x)$ with contained the operators in $f(x)$ and $f(x)$ and $f(x)$

$$
\langle x|\hat{O}|y\rangle = \int dp \langle x|p\rangle \, O_W\left(p, \frac{x+y}{2}\right) \langle p|y\rangle
$$
\n
$$
O_W(p,q) = \int dx \, e^{ipx} \langle q - \frac{x}{2}|\hat{O}(\hat{p},\hat{q})|q + \frac{x}{2}\rangle
$$

 $H_n(p,q) = \left(\frac{p^2 + \omega^2 q^2}{2}\right)$ 2 \setminus^n for $H_n(p,q) = \left(\frac{p+q}{q}\right)$ the models in \mathcal{L} and n \mathcal{L} and n $=$ 3 cases such symbols readily First example for

 \mathcal{L} in the configuration space for \mathcal{L}

Weyl symbol
$$
H_{2W}(p,q) = \left(\frac{p^2+q^2}{2}\right)^2 - \frac{1}{4}
$$
, $H_{3W}(p,q) = \left(\frac{p^2+q^2}{2}\right)^3 - \frac{5}{4}\left(\frac{p^2+q^2}{2}\right)$

the flow will generate also a dependence on time derivatives of q and p variables. This goes beyond the LHA but it is still compatible with the standard Hamiltonian approach

as long as one starts the flow at the UV with a derivatives-free bare Hamiltonian.

one-to-one correspondence with the Hamiltonian of the operator representation: the bare

Hamiltonian is just the Weyl symbol of the Hamiltonian operator. Let us remind that an

 α , and α and β , and β and β and α sum of symmetrized (in β

built by ladder operators. Rescaling the variables q = q′ **Heart Except EXCLUME ENGLULLER ENGLULLER FOR DETERMINISHED FOR PROPERTY PROBLEM TO THE PROBLEM TO THE ONE WITH USE** the effective Hamiltonian. $\begin{bmatrix} H_2 \ H_2 \end{bmatrix}$ $\begin{bmatrix} 1/2 \ 1/8 \end{bmatrix}$ $\begin{bmatrix} 0.49989 \ 0.19498 \end{bmatrix}$ $\begin{bmatrix} 0.49989 \ 0.19498 \end{bmatrix}$ completely described by full quantum effective Hamiltonian at k = 0. Such a task can be performed by numerically two energy levels, this might be unnecessary: it could be enough to truncate the LHA to From numerical evolution one gets Numerical error in the spectrum $\langle 0.1\%$ $\frac{H_3}{H_3}$ $\frac{3}{4}$ $\frac{0.749849}{0.749849}$

where the brain eigenstates of the brain eigenstates of the states of the states of the \mathcal{R}

We started by started by started flows as generated flows as generated flows as generated by equations (35) as

and (36) finding that singularities appear at nonvanishing values of k. This happens

integration of the flow equation for Hk, no singularity is met and the ground state and

the boundary conditions (which can be controlled by some nonlinear redefinitions of Hk)

"p² + q²

^k)1/² at the minimum. However

 $\frac{1}{2}$ \mathbf{r} polynomial in z \mathbf{r} (p2 + \mathbf{r} of finite order. Indeed if the bare Hamiltonian depends on \mathbf{r} on p and α on the LHA approximation also H_H can be shown to respect to res Diagonal cutoff schemes seem to work better. P is no clear pattern on the change of the precision of the results when increasing the order

operator. This is enough to deduce the whole energy spectrum for any positive integer n.

Hamiltonian is just the Weyl symbol of the Hamiltonian operator. Let us remind the Hamiltonian operator. Let us δ , can always be written as a sum of symmetrized (in p and δ

Oˆ = Oˆ^S +%

In order to reproduce such a spectrum by means of the RG flow equation, the first of the first \mathcal{L} step is to specify the initial condition for the integration of the flow. From the discussion of the flow of the flow. From the flow of the flow. From the discussion of the flow. From the discussion of the flow. From the d b^{max} powers of z goes to zero, a fact related to zero, a fact related to the vanishing of the terms α

The Hamiltonian operator is just the ⁿ-th power of #

The Hamiltonian $U(x,s) = n^h + s s^n$ has instead the same W **The Hamiltonian** $H_n(p,q) = p^n + a q^n$ has instead the same W Similar agreement. one-to-one correspondence with the Hamiltonian of the operator representation: the operator representation: the bare \mathcal{L}_max \mathbb{R}^n has instead the same Weyl symbol p and zero, and zero, a fact relation to p and p the vanishing of the fields in the field the bare Hamiltonian of the n = 2 model. If no expansion is performed, as in the numerical $\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$ $\sum_{n=1}^{\infty}$ has instead the same Weyl symbol The Hamiltonian $H_n(p,q) = p^n + a q^n$ has instead the same Weyl symbol is non-separable in p and q.

or because of numerical errors: typically we reached no more than two digit accuracy

Regge limit of strong interactions Pomeron-Odderon Reggeon Field Theory

The main physical motivation is the idea that QCD, in the high energy (Regge) limit and at large distances, can be described by an effective theory such as Reggeon Field Theory (RFT), with local fields and local interactions. $s \rightarrow \infty$

- Possible transition from QCD to the RFT regime:
	- BFKL physics: fundamental gluon (and quarks) organise themselves in composite fields (of reggeized gluons) giving as leading color singlet objects interacting Pomeron and Odderon, BFKL Pomeron ($J > 1$), Odderon ($J \approx 1$) and both $\alpha' \approx 0$

 $t \simeq 0$

- This should be at the "UV" boundary of RFT, below which (at larger distances) they may be considered approximately local with $J \approx 1$ and a non zero α' and described by Regge poles, as in old S-matrix analysis of strong interactions intrinsically non perturbative.
- The onset of such a transition should involve mainly perturbative physics.
- Here we investigate some features of RFT in 2 transverse dimensions

QCD in the Regge limit. plitude *N*(*x, y*; ⌧) and the *C*-odd (odderon exchange) amplitude *O*(*x, y*; ⌧) ζ also ζ and ζ and ζ

In early QCD times perturbative BFKL analysis found gluon reggeization, the Pomeron, as a composite state ψ of 2 reggeized gluons $\ddot{\ }$ (*^x ^z*)2(*^z ^y*)² [*N*(*x, ^z*; ⌧) + *^N*(*z, ^y*; ⌧) *^N*(*x, ^y*; ⌧) *N*(*x, z*; ⌧)*N*(*z, y*; ⌧) + *O*(*x, z*; ⌧)*O*(*z, y*; ⌧)] *,* (1) ψ Lipatov et al. (1977)

and later the Odderon (C,P odd), as a composite state χ of 3 reggeized gluons, solution of the DKD constitution in the largest near trivial expression is the la and fater the Odderon (C, P odd), as a composite state χ or 3 reggenzed gruons,
solution of the BKP equation in the lowest non trivial approximation. Bartels, Lipatov, G.P.V. (2000) *X*_{*x*}</sup> *Z*_{*x*}_{*x*} *z*_{*x*}</sup> *<i>n*_{*x*} *z*_{*x*} *z*_{*x}* Bartels, Lipatov, G.P.V. (2000)

Simple exchanges of such objects are corrected by interactions in presence of more reggeized gluons in the t channel which are necessary to unitarize the theory. non die general terms are associated to the triple possible points of the triple points of the t on are necessary to unitarize the theory.

Diagrams with reggeized gluons containing PPP and POO vertices: interactions are local in rapidity but non local in transverse space.

$$
\frac{\partial N}{\partial \tau} = KN - V_{PPP}NN + V_{POO}OO
$$

$$
\frac{\partial O}{\partial \tau} = KO - V_{OPO}(NO + ON)
$$

space vertices. We can write them symbolically in a more compact form

In the Region \mathcal{L} in the Region \mathcal{L} in the Region \mathcal{L} and also in Dipole/CGC/Wilson line approaches (NLLA to be checked) have shown that the evolution in the rapidity ⌧ for the *C*-even (pomeron exchange) am-

Approx, evolution in rapidity and are the pomeron and odderon fields respectively for suitable *X* and Approx. evolution in rapidity

h*T†*i*Y* where

Similar objects are found in other approaches to the Regge limit of QCD: CGC, Dipole/Wilson lines. gators integrated with the target impact factor. Note that physically the Former Regge amplitude receives negative contributions \mathbb{R}^n

RFT might appear at high energies (large rapidities) and large transverse distances.

Odderon recently in the news because of TOTEM measurements at LHC!

Strong interactions and old Regge theory In this section we briefly review the AGK strategy and its section we briefly review that $\mathbf{A}=\mathbf{A}+\mathbf{A}+\mathbf{B}$ ong interactions and old Regge theory will be concluded that the conclusion of the conclusion of the conclusion \sim the derivation and the study of the coupling of \sim \mathbf{a} possibility that the reggeons *i* and *j* form a composite $T_{\rm eff}$ and a multi-Region and a multi-Region and $T_{\rm eff}$ ze theory and the elastic scattering and the elastic scatter (Fig. 1), written as a Sommerfeld–Watson representation: $\ddot{\cdot}$ and the population to p $\ddot{\cdot}$ trons and old kegge theory

About half a century ago V.N. Gribov introduced phenomenologically the RFT. Starting point: Sommerfeld-Watson representation of the elastic scattering amplitudes. The original Age of the original Age paper starts from a multi-(Fig. 1), written as a Sommerfeld–Watson representation: the RFT. $n = \frac{1}{2}$, the matrix $\frac{1}{2}$ *Tally the RF₁</sub>* scally the RF₁ with ω = *J* − 1, $\frac{1}{2}$ and particle-pomeron couplings $\frac{1}{2}$ 2.1 The non-perturbative AGK rules (or more) reggeized gluons to virtual photons. t roduced phenomenologically the $RF1$. representation of the elastic scattering amplitudes. (Fig. 1), written as a Sommerfeld–Watson representation: **with NET** \blacksquare

$$
\mathcal{T}_{AB}(s,t) = \int \frac{d\omega}{2i} \xi(\omega) s^{1+\omega} \mathcal{F}(\omega,t).
$$

$$
\xi(\omega) = \frac{\tau - e^{-i\pi\omega}}{\sin \pi\omega}
$$

$$
\tau = \pm 1
$$

In this section we briefly review the AGK strategy and its

coupling of *n* Regge poles to the external particles, de-

pole contribution to the elastic scattering amplitude

(Fig. 1), written as a Sommerfeld–Watson representation:

noted by *Nn*(k*^j ,* ω). In general, they are functions of ω

\$

sin πω

= i ^e−ⁱ ^π

 \mathbb{R}^n

(or more) reggeized gluons to virtual photons.

ta

2

THE R. P. LEWIS CO., LANSING, MICH.

tial wave *F*(ω*, t*) has singularities in the complex ω-plane,

Fig.

tan

task is the derivation and the study of the coupling of four

^TAB(*s, t*) = ! ^d^ω

(or more) reggeized gluons to virtual photons.

tan

-

2i ^ξ(ω)*s*1+ω*F*(ω*, t*)*.* (1)

 $\mathbf{r} = \pm \mathbf{r}$

task is the derivation and the study of the study of $\tau = \pm 1$

tial wave *F*(ω*, t*) has singularities in the complex ω-plane,

2i ^ξ(ω)*s*1+^ω*F*(ω*, t*)*.* (1)

#

π

 $\frac{1}{2}$

more words about the origin of this formula. Following the

 $-$

• Regge pole description in the complex $\omega = J - 1$ plane with ω = *J* − 1, \mathcal{L}_{eff} plex $\omega = J - 1$ plan

2.1 The non-perturbative AGK rules

- The leading pole: even signatured Pomeron with vacuum quantum numbers, trajectory $\alpha(t) \simeq \alpha_0 + \alpha' t$ th vacu with vacuum quantum numbers, idea of Gribov, Pomeranchuk and Ter-Martirosian [12] one mediate state in the *t*-channel unitarity equation in the physical region of the process *^A* ⁺ *^A*¯ [→] *^B* ⁺ *^B*¯ (Fig. 2b). and the signature. The signature of \mathcal{L} The original AGK paper starts from a multi-Regge (Fig. 1), written as a Sommerfeld–Watson representation: sin πω ω + ¹−^τ
- Unitarity in the crossed (t-channel): multi pomeron states, branch-point singularities (Regge cuts) **b** α 1 **b** 1 **contract**) and **real-valued** particles before, particle point singularities point singularities amplitude (and the corresponding one below the \mathcal{A} *T* ron states, l \mathbf{r} ran^o $ch-p$ ₁ int si ngularities
- Analysis of experimental inclusive cross sections in the triple Regge region showed that a triple Pomeron interaction should be introduced. **wegge region
∂ ze cross sections in the triple Regge region**
- In the '70 it was conjectured that another pole with odd quantum numbers (P, C, τ) could exist, the so called Odderon with α(*0*) close to 1. sin πω
- The Pomeron RFT was found to be in the same universality class as directed percolation. Non perturbative FRG analysis give good results! **banels, Conneras, G.P. v.** (2010), Cardy (1980), Canet, et al. (2004(, Bartels, Contreras, G.P.V. (2016),

RFT with Pomeron and Odderon fields *<u>ith Pomeron and Odderon fields</u>* $\begin{array}{cccc} \text{+1} & \text{1} & \text{1} & \text{2} & \text{2} & \text{3} & \text{4} & \text{5} & \text{6} & \text{6} & \text{7} & \text{7} & \text{7} & \text{8} & \text{7} & \text{8} & \text{8} & \text{9} & \text{1} & \text$ +*i* ⇣ 58⁴ *†* ⁺ *†*⁴) + 59(*†*) ²(+ *†*) ⌘ +510*†* (² + *†*² 1.1 Signature factors and the set of the set o $1.1 W$ It is in the Demeration of the direction of the structure potential compared to the election of the election o to the pure Pomeron case. As described in Fig. 1, for the pure Pomeron case the pure Pomeron case the coupling It is written to note that displacement of the structure potential compared potential compared positive potential compared point in the east of the east of the east of the structure point in the east of the east of the eas d_{α} \ddot{r}_{α} d_{α} de <u>junior region</u>

51(*†*) ²(+ *†*) + ⁵² *†* (³ ⁺ *†*³) + 53*†* (³ ⁺ *†*³) + ⁵⁴ *† †*

²(+ *†*

real-valued for even powers of the Pomeron fields, whereas odd powers require in the Pomeron fields, whereas o
The Pomeron fields, whereas odd powers require imaginary require imaginary requirements of the Pomeron fields,

to the pure Pomeron case. As described in $[1]$, for the pure Pomeron case the pure Pomeron case the couplings are \mathcal{C}

energy has a minus sign which is obtained by requiring the triple pomeron coupling the triple Pomeron coupling

(² ⁺ *†*²

+510*†*) + 511*†*

²(+ *†*

Symmetries

+510*†*

+510*†*

+*i*

 \vert

in γ∗γ[∗] scattering

represent pomerons

Interactions are constrained by signature: conservation Reggeons have different signature factors, *i* + 58,800 *have enforcing against the costs,* **Example 3 Example 3** & **Represent** signature factors $\frac{1}{2}$ Symmetries Reggeons have different signature factors, t indict reggeon cut has discontinuity with overall sign from $-1H_j(G_j)$ Interactions are constrained by signature: conservation \overline{c} multi reggeon cut has discontinuity with overall sign from $-i\Pi_j(i\xi_j)$ Interactions are constrained by signature: const Express a constraint and the even-signature of the even-signature of the even-signature of the Pomeron exchange which is a set of the Pomeron exchange which is a set of the Pomeron exchange which is a set of the Pomeron ex real-valued for the Pomeron fields, whereas of the Pomeron fields, whereas of the Pomeron fields, whereas $\frac{1}{2}$ metries Reggeons have different signature factors multi-registed and domination in the matter of the contribution in the single state of $\prod_i (i \xi_i)$ The comparison with the original AGK paper allows us to community the outcome of the sign has the form of the form o $\frac{1}{2}$ sign from $-\frac{1}{I}$ *j* (i ξ_j)

) + 43(*†*

) + ⁵² *†*

(² *†* + *†*²

(² *†* + *†*²

)

² + *i*44*†*

(³ ⁺ *†*³

(² ⁺ *†*²

+*i*

⁺55(² *†*³

leads to special trigonometric factors in front of multi-pomeron cut contributions in the

has several consequences. First, because of signature conservation, t-channel states with the channel states with

result, all triple couplings are imaginary, except for the real-valued transition *P* ! *OO*.

) + 53*†*

58⁴ *†* ⁺ *†*⁴

) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (2.4) (

(³ ⁺ *†*³

+ *†*²

) (2.4)

momenta with ^q² ⁼ [−]*t*, and ^α(−k²

) + ⁵⁴ *†*

j=1

) (2.4)

 \mathbf{K}

π

^j cos ⁺

discontinuity in angular momentum of the partial wave

of (at least) two particles, and the complex angular mo-

mentum of the two particles is put equal to the trajectory

function of the Regge pole. The formula (3) contains the

-

-

) + 511*†*

*†*²

It is important to note the di↵erences in the structure of the e↵ective potential compared

This means, in particular, that the two Pomeron cut contribution to the Pomeron self-

energy has a minus sign which is obtained by requiring the triple Pomeron coupling to be

exceptions: the transitions *O* ! *OOO* and *P* ! *P* +*OO* are imaginary. This can be easily

 \mathcal{U} understood considering a contribution to such \mathcal{U}

)
1110
1111

² ⁺ *†*²

³) + 56(² *†*²

) + 59(*†*

the Regge pole trajectory function. The factor which de-

It is important to $\zeta = v$ (the end of the electrones in the e $\pi\omega$ (the equal compared points of $\pi\omega$ r_{r} (imaginary) Ω ddamas ζ ot Ω (mal) τ_{ω} (integrate), τ_{ω} conserversing $\pi\omega$ (row) $\sum_{k=1}^{\infty}$ to special trigonometric factors in the multi-pomeron cut contributions in the multi-political triputations in the multi-political triputations in the multi-political triputations in the multi-political tri $\pi\omega$ (1)**c** (1) $\pi\omega$ (1) π ω Pomeron: $\xi \simeq i$ (imaginary) , Odderon: $\xi \simeq -\frac{2}{\pi \omega}$ (real) *^j* (*i*⇠*^j*). For *|*⇡!*|* ⌧ 1 the pomeron multiple propagators, is given by *i ^j* (*i*⇠*^j*). For *|*⇡!*|* ⌧ 1 the pomeron T energy has a minus sign which is obtained by requiring the triple Pomeron coupling to be *^t*-channel unitarity equations: the *ⁿ*-Pomeron contribution comes with a factor (1)*n*1. **The Pomeron:** $\xi \simeq i$ (imaginary), Odderon: $\xi \simeq -\frac{2}{\pi\omega}$ (real) **include into property** power on $f \sim i$ (imaginary) Odderon: $f \sim$ $\zeta = e(\text{magmm})$, we calculate ζ

 $\mathcal{F}_{\mathcal{F}}$ for the gradient we add the following terms:

) + 59(*†*

with odderon fields was square is imaginary an such that *†* is real, i.e.

In the RFT language one is dealing with partial with partial wave amplitudes with particles which are also assumed to

real valued. For a single reggeon exchange (i.e. the reggeon propagator)

that for any region approximated by a linear trajectory one has \mathbb{R}^n

51(*†*

†

†

² + ⁴² *†*

)

) + 511*†*

) + 511*†*

*V*⁴ = 41(*†*

= *ei*⇡*/*4*z*, *†* = *ei*⇡*/*4*w*, with real *x, y, z, w*.

(² + *†*²

*V*⁵ = *i*

58⁴ *†* ⁺ *†*⁴

(² + *†*²

Fig. 1. Graphical representation of a multi-Regge poles con-

tribution to the elastic scattering and the elastic scattering amplitude. The zigzag lines and

= *ei*⇡*/*4*z*, *†* = *ei*⇡*/*4*w*, with real *x, y, z, w*.

Example 19 Solution Couplings can be real or imaginary! \mathcal{L} one considers a two region correction to the pomeron and the odd-based of \mathcal{L} eron, Souplings can be real or imaginary!
For the Souplings can be real or imaginary! purely in a can be For the Odderon the situation is di↵erent: the Odderon has negative signature. This Couplings can be real or imaginary!

rection to the Mueller–Navelet jet cross section formula

clusive cross sections, and in Sections, and 5 we turn to the section of the sections, and 5 we turn to the sec

075 [arXiv:1401.7431 [hep-ph]].

 \mathcal{A} and particle-pomeron couplings \mathcal{A}

- n Pomeron t-channel states induced by interactions gets a factor $(-1)^{n-1}$. f_{max} the newcher self-exercise position ende the pomeron sen energy is negative. The triple Pomeron coupling by convention is chosen imaginary. Quartic Pomeron couplings are real. Therefore the pomeron self energy is negative. • n Pomeron t-channel states induced by interactions gets a factor $(-1)^{n-1}$ F is the moment coupling the triple positive the normal coupling the triple F the pomeron sell energy is negative.
Pomeron-coupling by convention is chosen imaginary For the Odderon the situation is di↵erent: the Odderon has negative signature. This an odd number of odderons never mix with pure Pomeron channels. Second, the transition odderon state has *SOO* = +*i*(² Therefore the pomers
The triple **Pomeron** The triple Pomeron coupling by convention is chosen imaginary.
Quartic Pomeron couplings are road **Quartic Pomeron couplings are real.** - pomeron with *S^P* = *i*: two pomeron state has *SP P* = *i* and the two • n Pomeron t-channel states induced by interactions gets a factor $(-1)^{n-1}$ Therefore the nomeron self energy is negative • n Pomeron t-channel states induced by interactions gets a factor $(-1)^{n-1}$ Therefore the pomeron self energy is negative. This paper is organized as follows. In Sect. 2 we briefly As an example, the contribution of two even-signature negative compared to the single pole contribution. Equathe contribution of the *n*-reggeon *t*-channel state to the
- Odderon has negative signature: **P !** *P* \overline{P} $\overline{$ transition $P \to OO$ is real valued; transition $O \to OP$ is imaginary *<u>Prince interactions:</u>* most counting remain real, but result, all triple couplings are imaginary, except for the real-valued transition *P* ! *OO*. **Example 13** Ouartic interactions: most coupling remain real, but **P ! A ! Is real valued: the two-Odderon cut is positive (in contrast to the two-Odderon cut is positive** (in con cut), and there is no need for an imaginary coupling. On the other hand, the transition of the other hand, the result, all triple couplings are imaginary, except for the real-valued transition *P* ! *OO*. cussed for the evolution given in Eqs. (3). references Personnelle Quartic interactions: most coupling remain real, but *O* ! *O ! I* \sim *Odderon-Pomeron cut carries a minus sign. As a* **results** are in the real-valued triple couplings are imaginary, except for the real-valued transition *P* \overline{P} *OO... P* \overline{P} *<i>OO...*** ***P* \overline{P} *<i>OO... P* \overline{P} *<i>P* \overline{P} *P* \overline{P} *<i>P* In the sector of $P \to O\overline{O}$ is real-valued; all consider $O \to OP$ is imaginary Ω *Odderon*-bes peoptive signature: resultion *P* \cdot *OO* is real-valued transition *Q* \cdot *OP* is imaginary, In the sector of $\Gamma \to \overline{O}O$ is real-valued, and all situation $O \to \overline{O}O$ is imaginary • Odderon has negative signature: transition $P \to OQ$ is real valued; transition $O \to OP$ is imaginary **F** (Fig. 2). In the same way as in a usual unitarity intermass shell, in the reggeon unitarity integral the reggeons of the intermediate state are on shell in reggeon energy:

 $O \rightarrow OOO$ and $P \rightarrow P + OO$ have imaginary coupling Once the Odderon is included, again most quartic couplings ramain real, but there are two

exceptions: the transitions *O* ! *OOO* and *P* ! *P* +*OO* are imaginary. This can be easily

understood considering a contribution to such quartic vertices coming by the composition

 μ understood considering a contribution to such quartic vertices commonly by the composition of μ

es action for RFT e e un action has the form: *IDeal effective action for RFT , †* denote the Pomeron field, and for the Odderon we introduce the field *, †*. The

S[]+ *^k*

$$
\Gamma[\psi^{\dagger}, \psi, \chi^{\dagger}, \chi] = \int d^{D}x \, d\tau \left(Z_{P}(\frac{1}{2}\psi^{\dagger}\overleftrightarrow{\partial}_{\tau}\psi - \alpha'_{P}\psi^{\dagger}\nabla^{2}\psi) + Z_{O}(\frac{1}{2}\chi^{\dagger}\overleftrightarrow{\partial}_{\tau}\chi - \alpha'_{O}\chi^{\dagger}\nabla^{2}\chi) + V_{k}[\psi, \psi^{\dagger}, \chi, \chi^{\dagger}] \right)
$$

For the lowest truncation the e⊿ection of the form: • Allowed cubic interactions ✓ $\frac{1}{\sqrt{2}}$ $\overline{}$ \overline{A} *Howed cubic interactions*

, , †

e↵ective action has the form:

e↵ective action has the form:

*V*⁵ = *i*

+*i*⁴⁵ *†*

*V*³ = *µ^P †*

*V*⁴ = 41(*†*

d2*x* d⌧

+*Vk*[*, †*

⇣

)

*V*³ = *µ^P †*

+*i*

(² + *†*²

) + ⁵² *†*

µO†

For the quartic truncation we add the following terms:

²(+ *†*

+ *†*²

⁵⁸ (⁴ *†* ⁺ *†*⁴

,] = ^Z

d2*x* d⌧

+*Vk*[*, †*

[*†*

$$
V_3 = -\mu_P \psi^{\dagger} \psi + i\lambda \psi^{\dagger} (\psi + \psi^{\dagger}) \psi -
$$

$$
-\mu_O \chi^{\dagger} \chi + i\lambda_2 \chi^{\dagger} (\psi + \psi^{\dagger}) \chi + \lambda_3 (\psi^{\dagger} \chi^2 + {\chi^{\dagger}}^2 \psi)
$$

Bartels, Contreras, G.P.V. (2017),

2 The setup

ek[¯] =

Hamiltonian form

◆

†

. (2.1) . (2.1) . (2.1) . (2.2)

² ⁺ *†*²

² + *i*44*†*

(² ⁺ *†*²

\$

@⌧ ↵

)*.* (2.2)

)*.* (2.3)

(² ⁺ *†*²

) + ⁵⁴ *†*

)*.* (2.4)

² ⁺ *†*²

• Allowed quartic interactions \bullet *†* \mathbf{n} For the lowest quartic interactions $\frac{1}{\pi}$ $\frac{1}{\pi}$ $\frac{1}{\pi}$

 S in the following truncation has the following element terms: \mathcal{S} is the following element terms:

) + ⁵³ *†*

) $\frac{1}{\sqrt{2}}$

²(+ *†*

• …

(³ + *†*³

)

) + ⁵⁹ (*†*

 \mathbf{I}

]

For the lowest truncation takes the e \sim

µO†

$$
V_4 = \lambda_{41}(\psi\psi^{\dagger})^2 + \lambda_{42}\psi\psi^{\dagger}(\psi^2 + {\psi^{\dagger}}^2) + \lambda_{43}(\chi\chi^{\dagger})^2 + i\lambda_{44}\chi\chi^{\dagger}(\chi^2 + {\chi^{\dagger}}^2) + i\lambda_{45}\psi\psi^{\dagger}(\chi^2 + {\chi^{\dagger}}^2) + \lambda_{46}\psi\psi^{\dagger}\chi\chi^{\dagger} + \lambda_{47}\chi\chi^{\dagger}(\psi^2 + {\psi^{\dagger}}^2)
$$

(2)

1

D µ^k e

0

\$

 $\overline{1}$

^µ˙ *^k*

4

, , †

) + ⁵¹¹ *†*

(² *†* ⁺ *†*²

, (1.3)

\$

¯ *·*(¯)*Sk*[¯] (1.1)

d2*x* d⌧

51(*†* • States with even and odd Odderon number do not mix. *V*⁵ = *i* For the quartic truncation we add the following terms: *<i>k* and odd Odderon number do not mix.

2

 $S_{\rm{max}}$, the quintic truncation has the following electron has the following electron ϵ

couplings. This is a consequence of the even-signature of the Pomeron exchange which possible which which the

eWk['*,J*] =

leads to special trigonometric factors in front of multi-pomeron cut contributions in the

(³ + *†*³

⁺55(² *†*³ • The couplings λ_3 and similarly λ_{44} and λ_{45} play a special role: they are responsible **for the <u>change of the Odderon number</u>**) + 59(*†* for the change of the Odderon number (² + *†*²

2

 $\frac{1}{2}$ We shall study the RG flow equation for a generic potential expanded as polynomial weak field approximation. We shall study the RG flow equation for a generic potential expanded as polynomial in the ² + ⁴² *† weak field approximation.* (² + *†*² \mathbf{W}_{α} shall study the $\mathbf{D} \mathbf{C}$ flow equation to *v* eque ⇣ le shall study the RG flow equation for a generic potential expanded as polynomial in the ⁺55(² *†*³ ³) + ⁵⁶ (² *†*² contribution of the fluctuation of the path integral, the path integral, the generator of the connected *n*-point integral, the connected *n*-point integral, the connected *n*-point integral, the connected *n*-point integr

†

D µ() *eS*[]*Sk*['*,*⇠]*J·*⇠

(² ⁺ *†*²

 W_{α} shall consider a generic D dimensional transverse space but mainly we real-valued for even powers of the Pomeron fields, whereas odd powers require in a power state in a power state in We shall cancider a generic D dimensional transverse areae but meinly weak in D re share consider a generic D annohistonal transverse space out mainly work in D We shall consider a generic D dimensional transverse space but mainly work in D=2. **Similarly, the shall consider a generic D dimensional transverse space of** \mathbf{p} ²) + ⁵⁷ (² *†* e shall consider a generic D dimensional transverse space but mainly work in I) + ⁵⁹ (*†*

couplings. This is a consequence of the even-signature of the even-signature of the Pomeron exchange which which

leads to special trigonometric factors in front of multi-pomeron cut contributions in the

+⁵¹⁰ *†*

Z

) + ⁵⁴ *†*

RTF: construction of the flow equations constitute and the constitution of the derivation. The derivation of the deriv handle. Since we are interested in an analysis based on polynomial expansions of the potential in terms of the Pomeron fields, we find it more convenient to derive directly the flow equations for the polynomial coecients (couplings). α construction of the flow and the beta-functions of the couplings we find the coupling we find the inverse of \mathcal{L} t_1 construction of the flow coupling the control $\frac{1}{2}$ 0 *GO*(!*, q*) where the first term conserves *n*, the number of odderon pairs, the second one changes *n V* = *V n*=0 + *V [|]n|*=1 + *V [|]n|*=2 + *...* (2.5) $\frac{1}{2}$ cases of the flowing aon of the now equation Γ - construction of the flow equation cases is the following transition rules. *V* = *V n*=0 + *V [|]n|*=1 + *V [|]n|*=2 + *...* (2.5) where the construction of the now open.

|
|-
|-

For the potential we introduce

Next we introduce dimensionless variables. The field variables are rescaled as follows:

✓*D*

tential in terms of the Pomeron and Odderon fields, we find it more convenient to derive

exceptions: the transitions *O* ! *OOO* and *P* ! *P* +*OO* are imaginary. This can be easily \mathcal{L} understood considering a contribution to such \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L}

In this work we shall limit our shall limit our

of two triple ones. For the quintic part the 'exceptional' terms are in the second and fourth

In this work we shall limit ourself in analyzing the flow of the potential expanded around the origin (zero fields), i.e. we shall expansion. We shall expansion a weak field expansion. We shall expansion

 $\frac{1}{2}$ consequent the best formulation of the intension of the intention the following way: General strategy used here for a polynomial truncation of the potential. In this work we shall limit ourself in analyzing the flow of the potential expanded **1** • General strategy used here for a poly [(2) + R] ¹ = [(2) *free Vint*] for a polynomial truncation of the potential. (*Z^P* (*i*! + ↵⁰ If the potential *P* $\frac{1}{2}$ *OO <i>O OO OO* for the potential. olynomial truncation of the potential. by neglecting the quantum fluctuations in the fluctuations in the fluctuations in the fluctuation becomes: 'exceptional' couplings (e.g the transitions *P* ! *OO*, *O* ! *OOO*, or *P* ! *P* +*OO*), whereas (i) states with even and odderon numbers of $\mathbf{1}$. (ii) states will be labelled by the number of Odderon pairs, *n*. We assign a quantum by one etc. an the perturbative region of the transition of the perchanging α is a performance α

tential in terms of the Pomeron and Odderon fields, we find it more convenient to derive

This suggests to decompose the e↵ective potential into a sum terms *V* (*n*)

directly the flow equations for the polynomial coecients (couplings).

Pomeron couplings (e.g., the transition: Pomeron *to* four Odderons is imaginary).

directly the flow equations for the polynomial coecients (couplings).

transitions changing *n* by even numbers are 'normal' and have the same structure as pure

directly the flow equations for the polynomial coecients (couplings).

$$
\begin{aligned} [\Gamma^{(2)} + \mathbb{R}]^{-1} &= [\Gamma^{(2)}_{free} - V_{int}]^{-1} \\ &= G(\omega, q) + G(\omega, q)V_{int}G(\omega, q) + G(\omega, q)V_{int}G(\omega, q)V_{int}G(\omega, q) + \dots \end{aligned}
$$

This suggests to decompose the e↵ective potential into a sum terms *V* (*n*)

◆

!

$$
G(\omega,q)=\begin{pmatrix}G_P(\omega,q)\\ 0 & G_O(\omega,q)\end{pmatrix}\begin{matrix}G_P(\omega,q)\\ G_O(\omega,q)\end{matrix}=\begin{pmatrix}0\\ (Z_P(i\omega+\alpha'_Pq^2)+R_P-\mu_P)^{-1}&0\\ (Z_O(i\omega+\alpha'_Pq^2)+R_P-\mu_P)^{-1}&0\end{pmatrix}\begin{matrix}V^r_{\psi\psi}&V^r_{\psi\psi\psi}&V^r_{\psi\chi}+V^r_{\chi\chi}+V^r_{\psi\chi}+V^r_{\chi\chi}+V^r_{\psi\chi}+V^r_{\chi\chi}+V^r_{\psi\chi}
$$

regulator for the coarse-graining: $R_P(q^2) = Z_P\alpha$ $\alpha'_{P}(k^2 - q^2)\Theta(k^2 - q^2),$
 β' $(k^2 - q^2)\Theta(k^2 - q^2) = rZ \Omega'(k^2 - q^2)$ $P_{O}(q^2) = Z_{O} \alpha'_{O}(k^2)$ *^P ^q*2) + *^R^P ^µ^P*)¹ (*Z^P* (*i*! + ↵⁰ $\sum_{n=1}^{\infty} P_n(x) = \int_{0}^{\infty} P_n(x) dx$ $T_1 = \frac{1}{2}$ interaction matrix $\frac{1}{2}$ is defined from the e $\frac{1}{2}$ of the e $\frac{1}{2}$, after removal of the e $\frac{1}{2}$ 0 *V ^r* $R_O(q^2) = Z_O \alpha'_O(k^2)$ IR regulator for the coarse-graining: *† † ^V ^r V r* .
.
. CCCCA *.* (3.11) Finally we define the regulator matrix consisting of two block matrices. First we define $\Theta(k^2)$ $(-q^2)$ $r = \frac{\alpha'_O}{\alpha'_P}$ $R_P(q^2) = Z_P \alpha'_P (k^2 - q^2) \Theta(k^2 - q^2),$ ulator for the coarse-graining: $\frac{np(q)}{R_O(q^2) = Z_O\alpha'_O(k^2 - q^2)\Theta(k^2 - q^2)} = rZ_O\alpha'_P(k^2 - q^2)\Theta(k^2 - q^2)$ $r = \frac{1}{\alpha'_{P}}$ *O*_{*R*} $P(q^2) = Z_P \alpha'_P (k^2 - q^2) \Theta(k^2 - q^2),$ $V^2 \Theta(k^2 - q^2) = rZ_O \alpha_P'(k^2 - q^2) \Theta(k^2 - q^2)$ $r = \frac{\alpha_O'}{\alpha_P'}$ $R_P(q^2) = Z_P\alpha'_P(k^2 - q^2)\Theta(k^2 - q^2),$ including and $R_0(q^2) = Z_0 \alpha'_0 (k^2 - q^2) \Theta(k^2 - q^2) = r Z_0 \alpha'_P(k^2 - q^2) \Theta(k^2 - q^2)$ *^V*˜ ⁼ *^V* $R_P(q^2) = Z_P \alpha'_P(k^2 - q^2) \Theta(k^2 - q^2),$
 $R_P(q^2) = Z_P \alpha'_P(k^2 - q^2) \Theta(k^2 - q^2),$ $\mathcal{L}_{\text{U}(q)}$ is the dimensionless ratio of $\mathcal{L}_{\text{U}(q)}$

The interaction matrix *Vint* is derived from the e↵ective potential, after removal of the

found to be nonzero. As one of our results we shall see that the dynamics allows for a critical

Annalous dimensions:
$$
np = -\frac{1}{Z_P} \partial_t Z_P
$$
, $n_O = -\frac{1}{Z_O} \partial_t Z_O$ $\zeta_P = -\frac{1}{\alpha'_P} \partial_t \alpha'_P$, $\zeta_O = -\frac{1}{\alpha'_O} \partial_t \alpha'_O$

(*ZO*(*i*! + ↵⁰ *^Oq*2) + *^R^O ^µO*)¹ ⁰ \mathcal{V} ^{*i*} $\alpha'_{P}k^{D+2}$ *nensionless q* BBBB@ *† ^V ^r † † ^V ^r † ††* |
| 1 $\frac{\dot{\gamma}}{2}$ $\frac{\dot{\gamma}}{2}$ = α_p $e^{-D/2}$ $\psi, \tilde{\chi} =$ $\tilde{V} = \frac{Z_0^{1/2} k^{-D/2} \chi}{\tilde{V}}$ $\tilde{V} = \tilde{V} = \tilde{V}$ $\overline{}$ $\tilde{\psi} = 2$ $\frac{d^{1/2}}{P}$ $k^{-D/2}\psi, \quad \tilde{\chi} = Z$ $V = \frac{V}{\omega' k^{D+2}}$ Dimensionless quantities: $\tilde{\psi} = Z_P^{1/2} k^{-D/2} \psi$, $\tilde{\chi} = Z_O^{1/2} k^{-D/2} \chi$. $\tilde{V} = \frac{V}{\alpha'_{R} k}$ $\tilde{\psi} = Z_P^{1/2} k^{-D/2} \psi, \quad \tilde{\chi} = Z_O^{1/2} k^{-D/2} \chi. \qquad \qquad \tilde{V} = \frac{V}{\alpha'_{P} k^{D+2}}$ Finally, using Eq. (2.7) and (2.8), the couplings are rescaled in the following way:

@*t* ⁼ ¹

and

V r † ^V ^r † † ^V ^r † V r †† $\tilde{\lambda}$ For example:

<u>ssica</u>

Classical scaling:

–6–

 $|C|$

The momentum integral contained in the trace can be done in the same way as de-

scribed in [1]. The energy integral will be performed by complex integration. Unfortunately the analytic expression for the full flow of the potential is quite involved and discussed and discussed and di

consider more refined analysis in a future investigation. Therefore, for the derivation of

= *G*(!*, q*) + *G*(!*, q*)*VintG*(!*, q*) + *G*(!*, q*)*VintG*(!*, q*)*VintG*(!*, q*) + *...*(3.7)

where we have a strong strong with the strong strong strong with the strong strong strong strong strong strong

✓*D*

 $(2,2)$

^G^P (!*, q*) =

^GO(!*, q*) =

reggeon masses:

 $\frac{1}{2}$

(*ZO*(*i*! + ↵⁰

Vint =

$$
\hat{\mu}_P = \frac{\mu_P}{Z_P \alpha_P' k^2}, \quad \hat{\mu}_O = \frac{\mu_O}{Z_O \alpha_P' k^2},
$$
\nFor example:

\n
$$
\tilde{\lambda} = \frac{\lambda}{Z_P^{3/2} \alpha_P' k^2} k^{D/2}, \quad \tilde{\lambda}_{2,3} = \frac{\lambda_{2,3}}{Z_O Z_P^{1/2} \alpha_P' k^2} k^{D/2}
$$
\nClassical scaling:

\n
$$
(- (D + 2) + \zeta_P) \tilde{V} + \left(\frac{D}{2} + \frac{\eta_P}{2}\right) \left(\tilde{\psi} \frac{\partial \tilde{V}}{\partial \tilde{\psi}} + \tilde{\psi}^\dagger \frac{\partial \tilde{V}}{\partial \tilde{\psi}^\dagger}\right) + \left(\frac{D}{2} + \frac{\eta_O}{2}\right) \left(\tilde{\chi} \frac{\partial \tilde{V}}{\partial \tilde{\chi}} + \tilde{\chi}^\dagger \frac{\partial \tilde{V}}{\partial \tilde{\chi}^\dagger}\right)
$$

^µ˜*^P* ⁼ *^µ^P*

 \sim

^P (*k*² *^q*2)⇥(*k*² *^q*2)*,*

The scale *k* dependent regulator functions are chosen as follows:

ś (2) *^P P O OP* (2)

^P ↵⁰

1@*t*R*.* (3.1)

!

*^P ^k*² *, ^µ*˜*^O* ⁼ *^µ^O*

The trace on the rhs extends over a 4*x*4 matrix. The propagator matrix can be written

 \mathbb{R}^2

*^kD/*2*,* ˜2*,*³ ⁼ 2*,*³

, (3.2)

*^P k*²

*ZOZ*1*/*²

The scale *k* dependent regulator functions are chosen as follows:

|
|-

:

including anomalous dimensions (LPA')) at which *n* is conserved, i.e. all couplings which

|
|-

@*V*˜

。

¹ = [(2)

lines: in all these terms we either create or annihilate a pair of Odderons.

:

free Vint]

The signature-conservation rule, together with the appearance of these 'exceptional'

of two triple ones. For the quintic part the second and for the second and fourth α

(ii) states will be labelled by the number of Odderon pairs, *n*. We assign a quantum

Here we absorb the masses (intercepts minus one) into the free propagators:

The interaction matrix *Vint* is derived from the e↵ective potential, after removal of the

This applies, in particular, the coupling of the *P* ! *OO* transition.

V = *V n*=0 + *V [|]n|*=1 + *V [|]n|*=2 + *...* (2.5)

This choice implies that we introduce the dimensionless ratio \mathcal{L}_max

Z^P ↵⁰

= *G*(!*, q*) + *G*(!*, q*)*VintG*(!*, q*) + *G*(!*, q*)*VintG*(!*, q*)*VintG*(!*, q*) + *...*(3.7)

theory (as a fixed point of the flow in the local potential approximation (LPA), eventually

Pomeron couplings (e.g., the transition: Pomeron *to* four Odderons is imaginary).

This suggests to decompose the e↵ective potential into a sum terms *V* (*n*)

, (3.8)

*^P ^q*2) + *^R^P ^µ^P*)¹

*^Oq*2) + *^R^O ^µO*)¹

 $\frac{1}{2}$

*^ZOZ*1*/*²

, (3.2)

*kD/*2*.* (2.11)

!

(2.12)

^P ↵⁰

*^P k*²

^V [|]n|=1 ! ⁰*, ^V [|]n|*=2 ! ⁰*,* (2.6)

† @*V*˜

 $\ddot{}$

*^O ^kD/*2*.* (2.7)

,

^P . (2.10)

The trace on the rhs extends over a 4*x*4 matrix. The propagator matrix can be written

can be very only to be the physical relevant region. The physical region of the physical region. The beta function of the physical relevant region. The beta function of the physical relevant region. The beta function of th *Cubic truncation: beta functions* @(*i*!) *I*_c truncation $\frac{1}{2}$ ⌘*^O* ⇣*^O* ⁼ ¹ lim !!0*,q*!⁰ @ @*q*² *^I* (1*,*1) *^O* (!*, q*)*.* (3.37) + ²(3*A^P* + *AOr*)(*r µO*) (1 *^µ^P*)(1 + *^r ^µ^P ^µO*)³ ⁴2² \mathbf{u} unctions 1 *^V^k ^V^k † † Z*² *^k*!² + (*h^k* + *V^k †*)² ¹ hata function *iZk*! *^h^k ^V^k † ^V^k* ! $\frac{1}{2}$ *d*

lim !!0*,q*!⁰

V˙

and

I

*kk*²

^k = 2*Zk*↵

^O (!*, q*) (3.35)

Z *d*!*dDq*

2*A^P* ²

^Gk(!*, q*) = ⇣

^k(*q*² + *Rk*) = *Zk*↵

(2⇡)*D*+1 ✓(*k*² *^q*2)

²(*^r ^µO*)²

*Z*2

Zk↵⁰

(1 *^µ^P*)2(1 + *^r ^µ^P ^µO*)³ ⁴*AOr*2²

Performing the traces, the beta functions for dimensionless quantities are: @ \cos , the beta functions ces the beta functions for dimensionless quantiti \mathbf{H} $\overline{\mathbf{r}}$ and $\overline{\mathbf{r}}$ and $\overline{\mathbf{r}}$ and $\overline{\mathbf{r}}$ ons for dimensionless quantities are (1 *µ^P*)(1 + *r µ^P µO*)² P = 2000 P

$$
\mu_{\rm F} = (-2 + \eta_{\rm F} + \zeta_{\rm F})\mu_{\rm F} + 2A_{\rm P} \frac{\lambda_{\rm f}^2}{(1 - \mu_{\rm F})^2} - 2A_{\rm O}r \frac{\lambda_{\rm f}^2}{(r - \mu_{\rm O})^2}
$$
\n
$$
\mu_{\rm O} = (-2 + \eta_{\rm O} + \zeta_{\rm F})\mu_{\rm O} + 2(A_{\rm P} + A_{\rm O}r) \frac{\lambda_{\rm f}^2}{(1 + r - \mu_{\rm P} - \mu_{\rm O})^2}
$$
\n
$$
\lambda = (-2 + D/2 + \zeta_{\rm P} + \frac{3}{2}\eta_{\rm P})\lambda + 8A_{\rm P} \frac{\lambda}{(1 - \mu_{\rm P})^3} - 4A_{\rm O}r \frac{\lambda_{\rm 2}\lambda_{\rm f}^2}{(r - \mu_{\rm O})^3}
$$
\n
$$
\lambda_{\rm 2} = (-2 + D/2 + \zeta_{\rm P} + \frac{1}{2}\eta_{\rm P} + \eta_{\rm O})\lambda_{\rm 2}
$$
\n
$$
+ \frac{2\lambda\lambda_{2}^2(6A_{\rm P} + 5A_{\rm O}\tau) + \lambda\lambda_{2}^2(A_{\rm P} + A_{\rm O}\tau) - 4\lambda_{2}\lambda_{3}^2(A_{\rm P} + 2A_{\rm O}\tau)}{(1 + r - \mu_{\rm P} - \mu_{\rm O})^3} + \frac{4A_{\rm O}\lambda_{2}\lambda_{3}^2(1 - \mu_{\rm P})^2}{(1 - \mu_{\rm P})^2(1 + r - \mu_{\rm P} - \mu_{\rm O})^3} + \frac{4A_{\rm O}\lambda_{2}\lambda_{3}^2(1 - \mu_{\rm P})^2}{(1 - \mu_{\rm P})(1 + r - \mu_{\rm P} - \mu_{\rm O})^3} + \frac{2\lambda\lambda_{3}^2(3A_{\rm P} + 3A_{\rm O}\tau)(1 - \mu_{\rm P})}{(1 - \mu_{\rm P})(1 + r - \mu_{\rm P} - \mu_{\rm O})^3} + \frac{2\lambda\lambda_{3}^2(3A_{\rm P} + 2A_{\rm O}\tau)}{(1 - \mu_{\rm P})(1 + r - \mu_{\rm P} - \mu_{
$$

 $\mathcal{L}(\mathcal{L})$ is a point the lowest (cubic) truncation. For this approximation of the e $\mathcal{L}(\mathcal{L})$ tential, we keep on the rhs of (3.18) the terms with two and three V's. The *z*-integral is

^P (!*, q*) (3.34)

⌘*^O* ⁼ ¹

The anomalous dimensions are then given by:

and

Z^P ↵⁰

1

equation for *r* then becomes:

dI(11) *P*

 $\frac{1}{2}$ *O*

dI(11)

dI(11)

From the flow equations we obtain:

*dq*² ⁼ *^N^D*

*dq*² ⁼ *^N^D*

(1*,*1)

dz = 2*N^D*

dz = 2*N^D*

⌘*^P* ⁼ ¹

 n depend upon the truncation since only cubic couplings are involved): n

^O (!*, q*) = 2

^D(1 *^µ^P*)³ ⁺

(!*, q*)*†*(!*, q*)

⌘*^O* ⁺ ⇣*^O* ⁼ ⁴*ND*²

lim !!0*,q*!⁰

I (1*,*1)

> $\frac{1}{2}$ $A_P = N_D A_D(\eta_P, \zeta_P), A_O = N_D A_D(\eta_O, \zeta_O).$ $A_P = N_D A_D(\eta_P, \zeta_P), \ \ A_O = N_D A_D(\eta_O, \zeta_O).$ $\Omega = N_D A_L$ rent de la partie d
La partie de la par

(1 + *r µ^P µO*)³

*^kk*² + *V^k †*

^k(✓(*k*² *^q*2)*k*² ⁺ ✓(*q*² *^k*2)*q*2) (2.26)

^k!² + (*h^k* + *V^k †*)² *V^k V^k † †*

¹ ⌘*k*+⇣*^k*

³(1 *^µ^P*)²

*k*2)

. (2.28)

(1 *^µ^P*)2(1 + *^r ^µ^P ^µO*)² *.* (3.21)

N^D (2.29)

. (2.28)

(2.30)

, (2.31)

. (2.33)

, (2.31)

. (2.33)

D

(1 *µO*)2(1 + *r µ^P µO*)³

= (2.27)

(*r µO*)(1 + *r µ^P µO*)³

³(*A^P* + 3*AOr*)(1 *µ^P*)

² (1 *^q*²

$$
N_D = \frac{2}{\sqrt{4\pi}^D \Gamma(D/2)}
$$

\n
$$
\lambda_2 \lambda_3^2 (1 - \mu_P)^2
$$

\n
$$
A_D(\eta_k, \zeta_k) = \frac{1}{D} - \frac{\eta_k + \zeta_k}{D(D+2)}
$$

Similarly, one can find the anomalous dimensions (from the flow of 2-point functions): $\overline{}$ and $\overline{}$ an Similarly and open find the enemalous dimens

$$
\eta_P = -\frac{2A_P\lambda^2}{(1-\mu_P)^3} + \frac{2A_Or\lambda_3^2}{(r-\mu_O)^3} \qquad \eta_P + \zeta_P = -\frac{N_D\lambda^2}{D(1-\mu_P)^3} + \frac{N_Dr^2\lambda_3^2}{D(r-\mu_O)^3}
$$

$$
\eta_O = -\frac{4(A_P + A_Or)\lambda_2^2}{(1+r-\mu_P-\mu_O)^3} \qquad \eta_O + \zeta_O = -\frac{4N_D\lambda_2^2}{D(1+r-\mu_P-\mu_O)^3}.
$$

^D(*^r ^µO*)³ (3.44)

| ⁼ *†*==*†*=0*.* (3.25)

itself depends upon the cuto↵ parameter *k* and therefore has its own beta function. The

^D(1 + *^r ^µ^P ^µO*)³ *.* (3.45)

Perturbation theory: ϵ -expansion: $D=4-\epsilon$ points of the cubic truncation for continuos dimensions (0 *<D<* 4). <u>tion</u> the ✓*d* Dry ◆ \cdot ϵ - \overline{r} (4⇡)*d/*2[1+*d/*2] 1 *e*ory: €-expansion: D=

Zk()(@)

✓*d*

2

Critical theory (fixed point): perturbative one loop results: $\mathbf{1}_{\text{theorem}}$ ($\mathbf{3}_{\text{theorem}}$), point), and $\mathbf{4}_{\text{theorem}}$ and $\mathbf{1}_{\text{theorem}}$ months. *real theory (fixed point): perturbative one loop results:*

$$
\mu_P = \frac{\epsilon}{12}, \quad \lambda^2 = \frac{8\pi^2}{3}\epsilon, \quad \eta_P = -\frac{\epsilon}{6}, \quad \zeta_P = \zeta_O = \frac{\epsilon}{12},
$$
\n
$$
\mu_O = \frac{95 + 17\sqrt{33}}{2304}\epsilon, \quad \lambda_2^2 = \frac{23\sqrt{6} + 11\sqrt{22}}{48}\epsilon, \quad \lambda_3 = 0, \quad \eta_O = -\frac{7 + \sqrt{33}}{72}\epsilon, \quad r = \frac{3}{16}(\sqrt{33} - 1)
$$

dimension of the system and the system. In this section we show the results of the results of an analysis of the theory of t

close to the critical dimension (*D* = 4 ✏) at one loop, restricted to the cubic truncation only. Such an analysis can help to identify a possible contribution of the system of the system which system which the system which havior of the system of the system which havior of the system which havior of the system w

2

˜ *v*˜⁰

^k +

*X*1*,k*()(@2)

4

1

(4⇡)*d/*2[1+*d/*2]

*X*2*,k*()(@)

1

1+˜*v*00

⁴ ⁺ *···*)

² +

そ

1
11

Critical exponents: two relevant directions exponents:

^Vk() + ¹

^k() = *d v*˜*^k* +

2

4 Numerical results of the state of the

k[˜] = ^Z

4.1 Search for fixed points

$$
\alpha_1 = -2 + \frac{\epsilon}{4} \rightarrow \nu_P = \frac{1}{2} + \frac{\epsilon}{16}
$$

$$
\alpha_2 = -2 + \frac{\epsilon}{12} \rightarrow \nu_O = \frac{1}{2} + \frac{\epsilon}{48}.
$$

The coupling of the changing Odderon number operator is zero! \overline{P} OD transition present in perturbative OCD is irrelevant The $P \rightarrow OO$ transition present in perturbative QCD is irrelevant and disappears. Suppression of high mass diffractive scattering processes. 2 48
anging Odderon number operator is zerol contribution of the fluctuation modes in the path integral, the generator of the connected *n*-point ne coupling of the changing Guderon humber operator is zero: Γ and $\Gamma \rightarrow 0$ and Γ and Γ and Γ and Γ is the presence of an infrared regulator which controls the presence of an infrared regulator Γ suppression of ingli mass unifactive scattering processes.

Demense accteria not offered de the pure expected of the Oddenon The Pomeron sector is not affected by the presence of the Odderon!

These qualitative features are maintained at non perturbative level!

number, and the direction in parameter space which contains the operator breaking such as a space which contains the operator breaking such as a space which contains the operator breaking such as a space which contains the

Non perturbative analysis in $D=2$

Explicit analysis at order 3,4,5 of the fixed points seems to show that the interactions changing the Odderon number are absent in the critical theory.

We perform the analysis of the fixed point up to order 9, neglecting (apart in r) the anomalous dimensions.

results obtained for the pure Pomeron case, where we have found a fixed point with on

starting at *k* 6= 0 at a generic value in the parameter space of the e↵ective potential (not

are the same as in the same as in the pure Pomeron case at the corresponding order. There exist three exists three ex eigenvectors which spans the three three three three subspace α couplings α , β , 44, 45. They are the three thre for di↵erent orders *n* of the polynomial (3 *n* 9). The two negative leading eigenvalues Anomalous dimensions (cubic truncation estimate, close to ϵ -expansion result): and Odderon fields (left panel). We report also the value of a third negative eigenvalue of a third negative e lowe dimensions (orbit two ortins ortine of and in other to Communication or al was annensions (caole trancation estimate, close to e-expansion result Anomalous dimensions (cubic truncation estimate, close to ϵ serve from Monte Carlo analysis in the pure Pomeron sector. This generalizes the previous theory is generalized for anomalous dimensions (cubic truncation estimulation of \mathbb{R}^n

 $\rho \rho$ is part of the 10-dimensional critical subspace is part of the 10-dimensional critical subspace is part of the 10-dimensional critical subspace. The 10-dimensional critical subspace is ρ and ρ and ρ and \r $\eta_P \simeq -0.33, \eta_O \simeq -0.35 \text{ and } \zeta_P = \zeta_O \simeq +0.17$ serve from $\eta_P \simeq -0.33$, $\eta_O \simeq -0.35$ and ζ_P

Conclusions and outlook

- Functional renormalization group is a powerful tool not yet fully exploited to study both critical and off-critical QFTs.
- It can be used both at perturbative and non perturbative (wilsonian) level
- In perturbation theory it is possible to directly compare or complement results with ones from CFT techniques (conformal universal data).
- At non perturbative level one has scheme dependent exact RG flow equations.
- Main problem: choice of truncations and approximations! Still new ideas are needed for a systematic control of the convergence.
- Gauge theories still harder to investigate at accurate level
- In many cases gives results at the level of montecarlo analysis for strongly interacting theories.
- At theoretical level tool to study the (geometry of) theory space of QFTs