An overview of the lattice approach to strongly coupled quantum field theory

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Outline

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A simple question

- What is this?
- The picture above, based on quark-model intuition, is actually a misleading one
- A more accurate cartoon of the proton would look like this
- The proton is *radically* different from positronium

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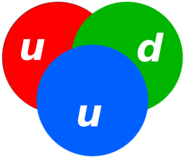

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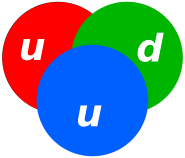

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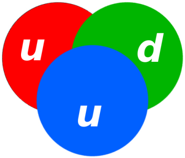

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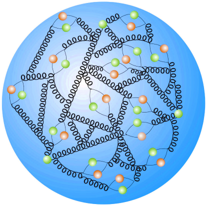

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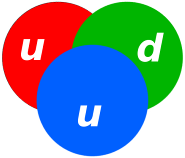

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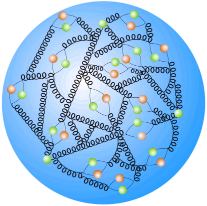

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Why do we need to the strong interaction on a lattice?

- In the Standard Model, the phenomenology of the strong interactions arises from the mathematical properties of quantum chromodynamics (QCD), a non-Abelian gauge theory which is *not* broken
- The QCD β-function implies that the strong interactions can be treated perturbatively for large transferred momenta
- Conversely, the physical α_s coupling becomes large at low energies
- The spectrum of the lightest hadronic states is determined by phenomena of *non-perturbative* nature:
  - Confinement of colored elementary degrees of freedom into color singlet states
  - Dynamical breaking of chiral symmetry
- The regularization on a spacetime lattice **[K. G. Wilson, 1974]** provides *the* mathematically rigorous, gauge-invariant definition of QCD at the non-perturbative level

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Basic ideas

- Regularize the path integrals by discretizing the theory on a Euclidean spacetime lattice of spacing a
- Define gauge and matter fields on lattice elements, build a gauge-invariant lattice action S_{E} and lattice observables
- The continuum Euclidean QCD action is recovered for $a \to 0$

$$S_{\mathrm{E}} = \int \mathrm{d}^4 x \left\{ \frac{1}{2} \mathrm{Tr} \left(F_{\mu\nu} F_{\mu\nu} \right) + \sum_f \overline{\psi}_f(x) \left(\gamma_\mu D_\mu + m_f \right) \psi_f(x) \right\} \cdot [1 + O(a)]$$

- A gauge-invariant, non-perturbative regularization
- Suitable for Monte Carlo integration: sample configuration space according to a *statistical weight* proportional to $\exp(-S_{\mathrm{E}})$, compute expectation values

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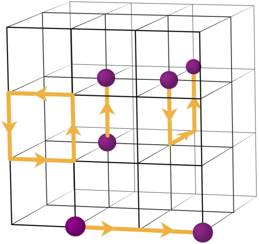

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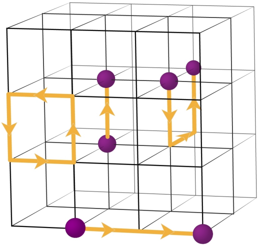

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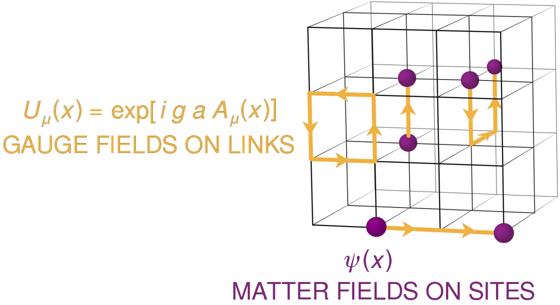

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$$S_{\mathrm{E}} = -\frac{1}{g^2} \sum_x \sum_{\mu,\nu} \mathrm{Tr} \left(U_\mu(x) U_\nu(x+a\hat{\mu}) U_\mu^\dagger(x+a\hat{\nu}) U_\nu^\dagger(x) \right) + \sum_{x,y,f} a^4 \overline{\psi}_f(x) M_{x,y}^f \psi_f(y)$$

$$M_{x,y}^f = m\delta_{x,y} - \frac{1}{2a} \sum_\mu \left[(r-\gamma_\mu) U_\mu(x) \delta_{x+a\hat{\mu},y} + (r+\gamma_\mu) U_\mu^\dagger(y) \delta_{x-a\hat{\mu},y} \right]$$

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$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}\psi(x) \mathrm{d}\overline{\psi}(x) \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S_{\mathrm{E}})}{\int \prod \mathrm{d}\psi(x) \mathrm{d}\overline{\psi}(x) \prod \mathrm{d}U_\mu(x) \exp(-S_{\mathrm{E}})}$$
$$= \frac{\int \prod \mathrm{d}U_\mu(x) \mathcal{O} \left(\prod_f \det M^f \right) \exp(-S_{\mathrm{E}}^{\mathrm{YM}})}{\int \prod \mathrm{d}U_\mu(x) \left(\prod_f \det M^f \right) \exp(-S_{\mathrm{E}}^{\mathrm{YM}})}$$

Observations

- The lattice introduces a finite momentum cutoff $O(1/a)$
- Gauge invariance is explicitly preserved *at all a*
- At finite a, Lorentz-Poincaré symmetries are broken down to discrete subgroups
- The lattice spacing a has no physical meaning: physical results obtained only in the continumm limit $a \to 0$
- At the quantum level, the continuum theory is *a good low-energy effective theory* for the lattice theory
- Required separation of scales:

$$1/L \ll \Lambda \ll 1/a,$$

  with L the linear extent of the lattice and Λ the scale of phenomena under consideration
- Euclidean formulation: Monte Carlo estimate of $\langle \mathcal{O} \rangle$ made possible by a *real positive* statistical weight proportional to $\exp(-S_E)$

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Scale setting

- What is the value of a, for a given set of parameters of the lattice theory?
- Scale setting: match a low-energy lattice observable to its continuum value
  - ★ Example in purely gluonic $SU(N)$ Yang-Mills theory: confining static potential from large Wilson loops $\mathcal{W}(r, L)$

$$\langle \mathcal{W}(r, L) \rangle \propto \exp\left(-\sigma r L\right) = \exp\left(-\sigma a^2 \cdot \frac{r}{a} \cdot \frac{L}{a}\right)$$

  - ★ Fit σa^2 from simulation results
  - ★ Deduce a using $\sigma = (440\text{MeV})^2$ and $197~\text{MeV} \simeq 1~\text{fm}^{-1}$
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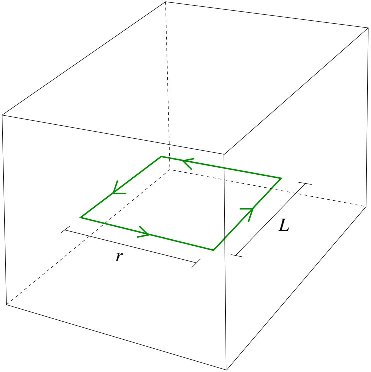

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Debunking some common misconceptions

- "Lattice QCD is only *an approximation* of QCD"
- "The results depend on the *details* of your discretization"
- "You can never recover the correct rotational and translational *symmetries* of the original continuum theory"
- "You always have undesired additional quark species (*doublers*)"
- "It only works / it is only defined at *strong* coupling"
- "It is numerically untractable: you can never be able to deal with those large Dirac operators / you are bound to neglect quark dynamics (*quenched approximation*)"

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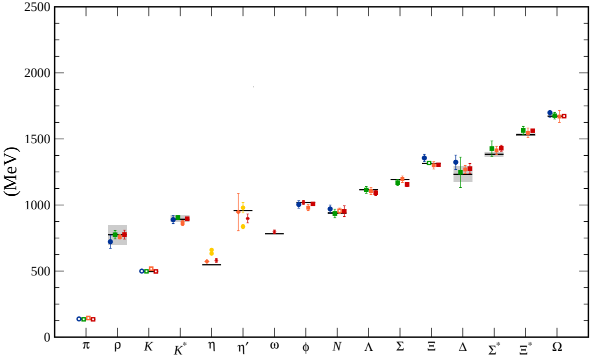
  - ★ Thanks to computer-power and algorithmic progress, quenched lattice QCD calculations are now mostly *obsolete*

Outline

Hadron spectrum

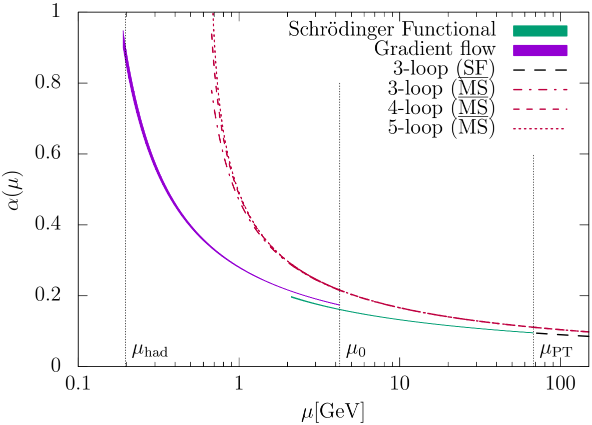
The calculation of the hadron spectrum from the first principles of QCD has always been a major motivation for lattice studies



Compilation of lattice results for the hadron spectrum obtained by various collaborations, adapted from **[A. Kronfeld, 2012]**. Open symbols denote the masses used to set the lattice parameters. Horizontal bars denote experimental masses, and gray boxes indicate experimental widths

QCD running coupling

Lattice calculations allow to compute the $\alpha_s = g_s^2/(4\pi)$ running coupling over a broad range of transferred momenta μ, from the perturbative down to the hadronic regime



Lattice results for α_s from the ALPHA Collaboration **[M. Bruno et al., 2017]**

Chiral dynamics and light-quark masses

- Because of color confinement, quark masses are not directly accessible to experiments
- For the lightest quark flavors (u, d, and possibly s), the highly non-trivial, strongly coupled dynamics of QCD at low energies implies that the origin of hadron masses *is not* from the constituent (valence) quark masses
- Chiral effective theory **[S. Weinberg, 1979] [J. Gasser and H. Leutwyler, 1984]** provides an elegant description of the low-energy physics of QCD with n_f (nearly) massless quark flavors, in terms of its (approximate) global chiral symmetry
- A low-energy effective theory, encoding the non-trivial dynamics of microscopic origin into an infinite set of *low-energy constants*: Σ (quark condensate), F ("pion" decay constant), ...
- These constants can be directly related to the masses of quarks and of hadrons, e.g. $m_\pi^2 F^2 = (m_u + m_d)\langle \overline{q} q \rangle$ **[M. Gell-Mann, R. J. Oakes, and B. Renner, 1968]**
- Lattice calculations of these low-energy constants allow to determine the light quark masses **[S. Aoki et al., 2013]**

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- Lattice studies of heavy quark flavors (c and b) are often based on heavy-quark effective theories, or on expansions around the non-relativistic limit
- Quantities of particular phenomenological interest include the spectrum of charmonium ($c\overline{c}$) states
- Semileptonic decays of B and D mesons (e.g. $B \to \pi l \overline{\nu}_l$), providing information on elements of the Cabibbo-Kobayashi-Maskawa, can be studied on the lattice by means of matrix elements of appropriate currents between the desired initial and final states, e.g. $\langle \pi(p) | V_\mu(q) | B(p_B) \rangle$ **[J. Laiho, E. Lunghi, and R. S. Van de Water, 2009]**

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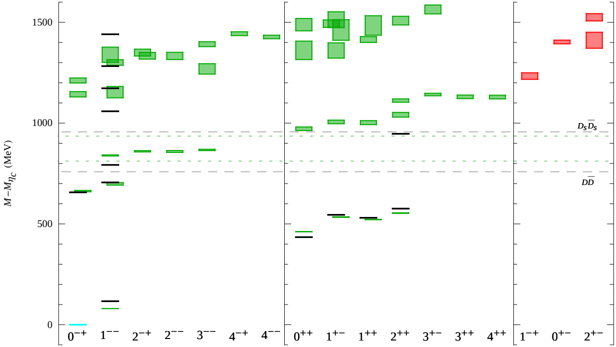
Charmonium spectrum from a lattice calculation by the Hadron Spectrum Collaboration **[L. Liu et al., 2012]**, showing the differences with respect to the ground-state η_c mass, whose experimental value is $M_{\eta_c} = 2983.6 \pm 0.7$ MeV



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QCD at high temperature

- At temperatures $T \gtrsim 160$ MeV, ordinary hadronic matter undergoes a crossover to a deconfined and chirally restored phase: the quark-gluon plasma
- Lattice results for equilibrium-thermodynamics quantities are consistent with the expected behavior in the low- and high-temperature limits, and interpolate smoothly between them
- The $\langle \overline{q} q \rangle$ quark condensate is found to drop with the temperature, in agreement with the restoration of $\mathrm{SU_A}(n_f)$ symmetry **[S. Borsányi et al., 2010] [A. Bazavov et al., 2011]**
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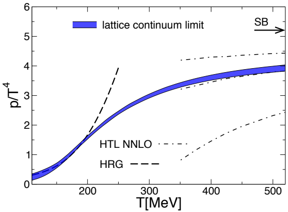

QCD pressure, as a function of temperature **[S. Borsányi et al., 2013]**

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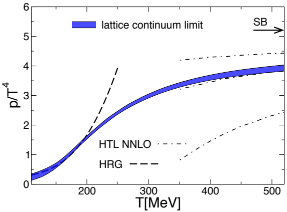

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Main features:

- ★ Confining phase: Consistency with hadron-resonance-gas model
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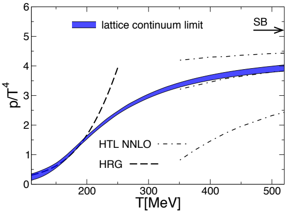

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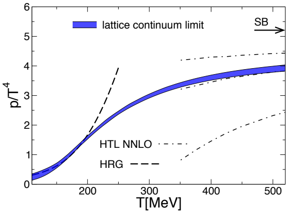

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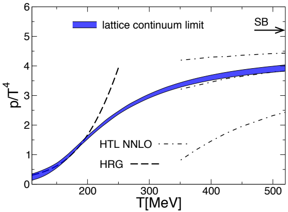

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Nuclear physics from lattice QCD

Recent works are studying (light) nuclei with lattice QCD **[M. J. Savage, 2011]**

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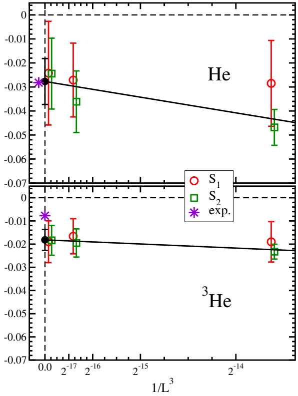

Binding energies of He and ^{3}He from a quenched lattice calculation at $a \simeq 0.128$ fm, with results displayed in units of $1/a \simeq 1540$ MeV **[T. Yamazaki et al., 2009]**

Applications beyond QCD

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Outline

Concluding remarks

- The regularization on a Euclidean spacetime lattice provides the mathematically rigorous gauge-invariant, non-perturbative definition of QCD
- The continuum QCD arises as a low-energy effective description of the lattice theory, in the limit in which the (unphysical) lattice cutoff $O(1/a)$ is sent to infinity
- Monte Carlo calculations on the lattice are now at a mature stage of development, and are providing first-principle QCD predictions for many physical quantities of experimental relevance
- Lattice calculations are also being generalized to challenging non-perturbative problems for physics beyond QCD
- Some important problems, however, are still unsolved—and waiting for your involvement
  - ★ "Sign problem" for QCD at finite baryon density
  - ★ Real-time dynamics from the lattice
  - ★ Theoretical formulation of chiral non-Abelian gauge theories (e.g. the electroweak theory of the Standard Model) on the lattice

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