Percolation and Cascading in a Brain Network of Networks

Hernán Makse
Physics Department
City College of New York

Jose Soares de Andrade (physics) - Brazil
Santiago Canals (neuro)- Alicante
Mariano Sigman (neuro)- Buenos Aires
Flaviano Morone (physics) CCNY
Lucas Parra (neuro) - CCNY
Xavier Gabaix (economics) - NYU
Fredrik Liljeros (sociology) - Stockholm

Capri, August 2015

OUTLINE: two percolation conundra and one application

1. Brain Conundrum 1: The “binding problem” in brain networks

Percolation of information flow in brain networks: Gallos, Makse, Sigman, PNAS (2012)

2. Brain Conundrum 2: Vulnerability to cascades of failure in a brain network of networks

Percolation of NoN: Reis, Canals, Andrade, Sigman, Makse, Nat. Phys. (2014)

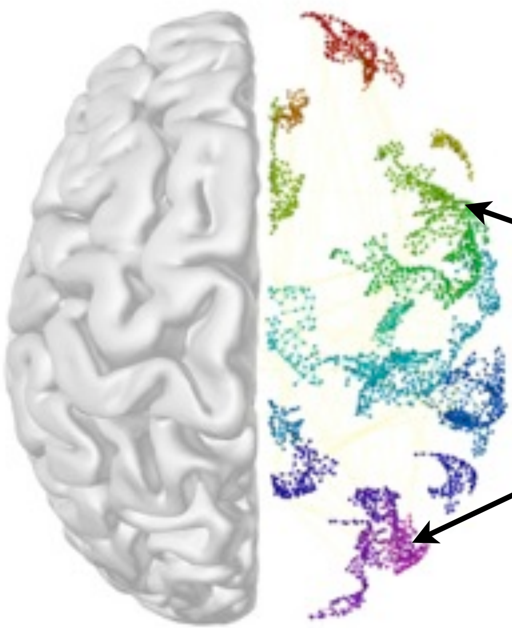
Optimal Percolation: Morone, Makse, Nature (2015)

3. Application: Emergence of “engagement” in eye-tracking and homophily from neural correlations.

Brain conundrum 1: Binding Problem

Brain modules ought to be sufficiently independent to guarantee functional specialization and sufficiently connected to bind multiple processors for efficient information transfer for, for instance, unitary perception (ie, visual areas analyze simultaneously form, color, motion, etc)

Segregation versus integration at the network level



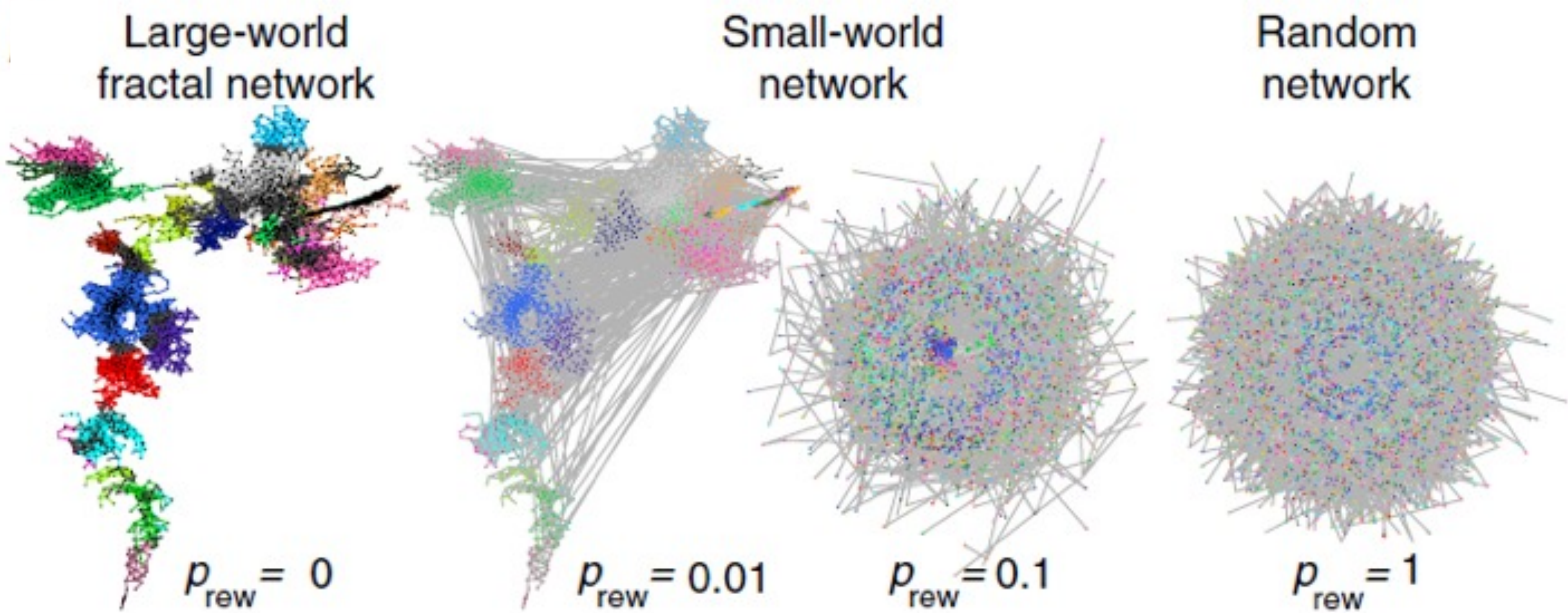
Problem of any information processing system:

Network of Networks

Prevailing model in neuroscience: Small-world network model

However, there is intrinsic tension between shortcuts generating small-worlds and the persistence of modularity; a global property unrelated to local clustering

Small-world destroys modularity



Watts-Strogatz small world networks

Watts, Strogatz, Nature, 1998

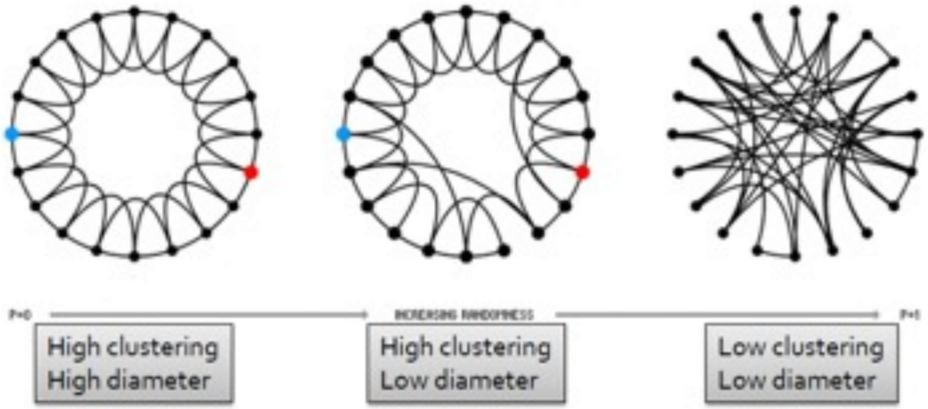
Start with a lattice. Rewire a fraction p of links to form a random graph

$$< l > \sim \ln N$$

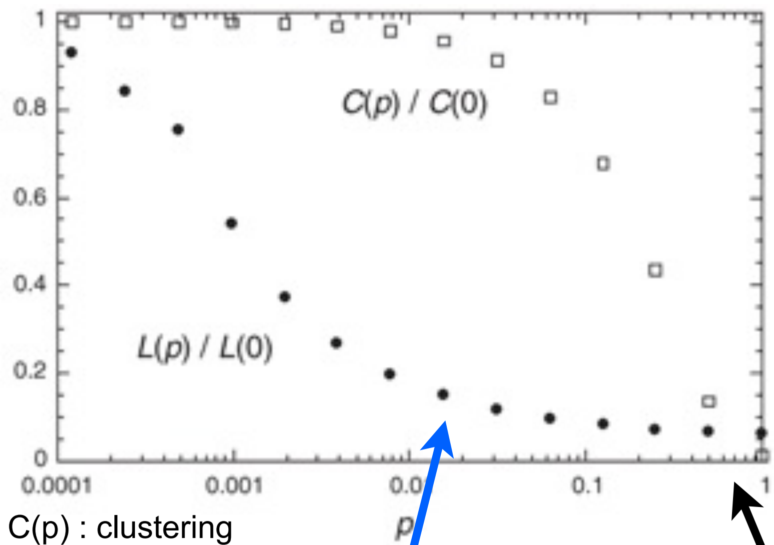
Big world: lattice

Small world

Random network



- Rewiring allows to interpolate between regular lattice and a random graph



C(p) : clustering coefficient

L(p) : average path length

Random network

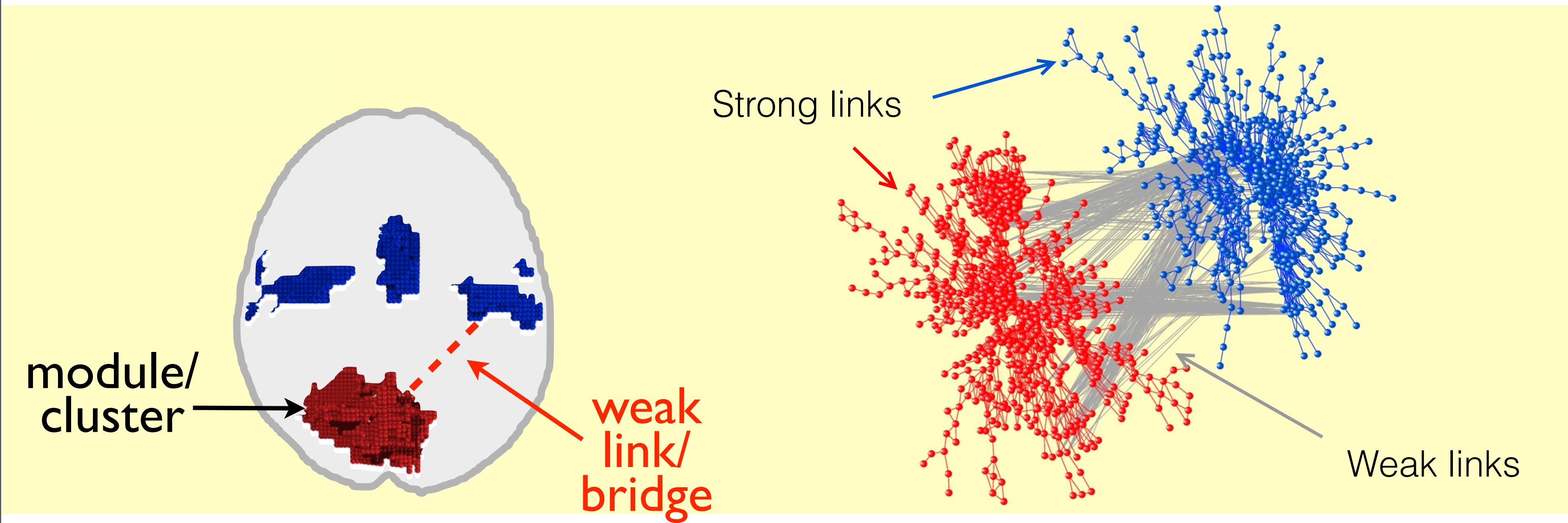
Small world: short path but high clustering

Six degree of separation

Our hypothesis: strength of weak links

Gallos, Makse, Sigman, PNAS 2012

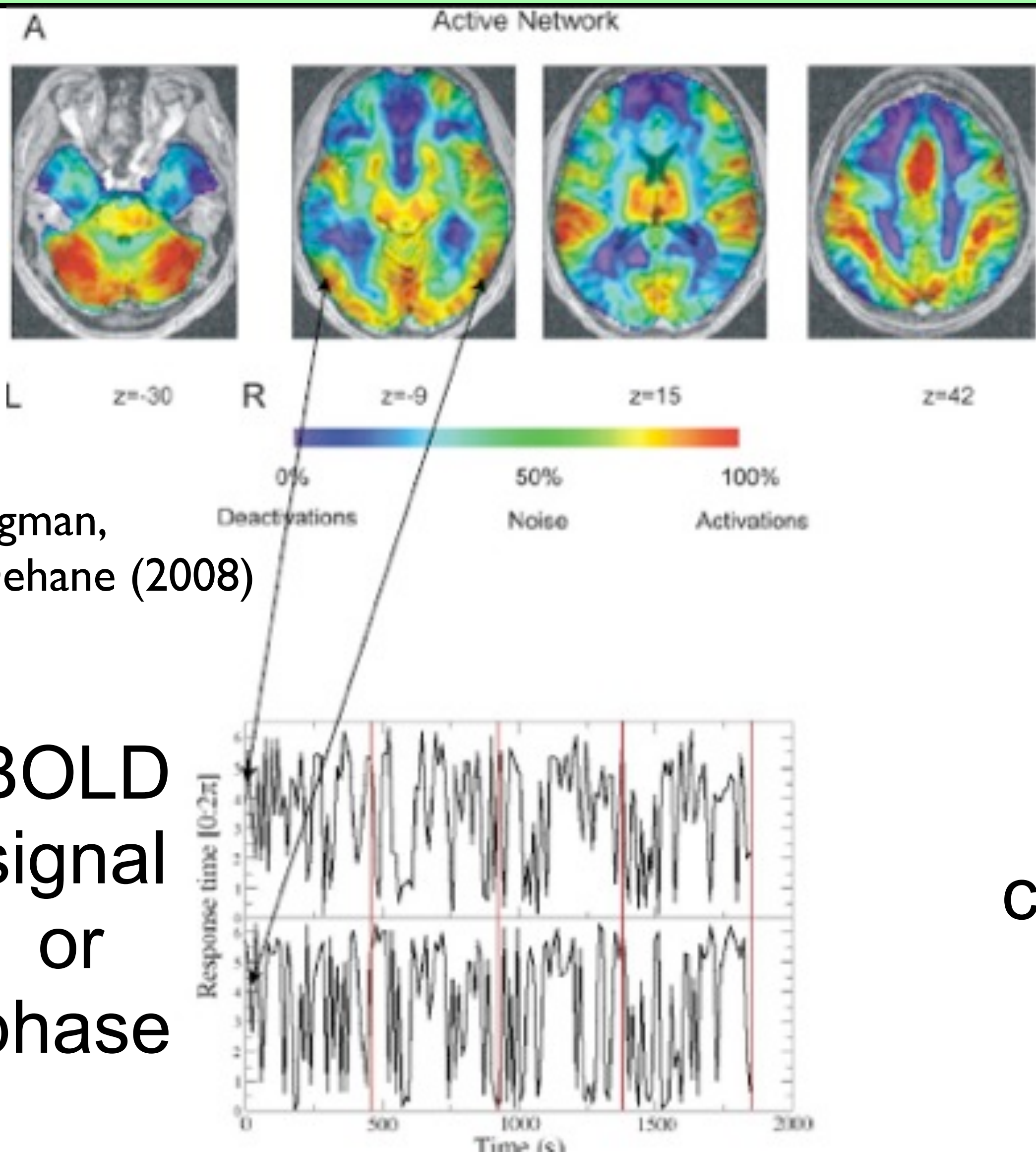
Inspired by Granovetter paradoxical social theory
"Strength of weak ties" (1973)



Strong links form a highly modular non-small world topology in a sea of weak links

Building a functional brain network from fMRI in dual task: visual + auditory

Resting state network: Raichle
Eguiluz,et al. PRL (2005)



Sigman,
Dehane (2008)

BOLD
signal
or
phase

Correlation between two voxels i, j:

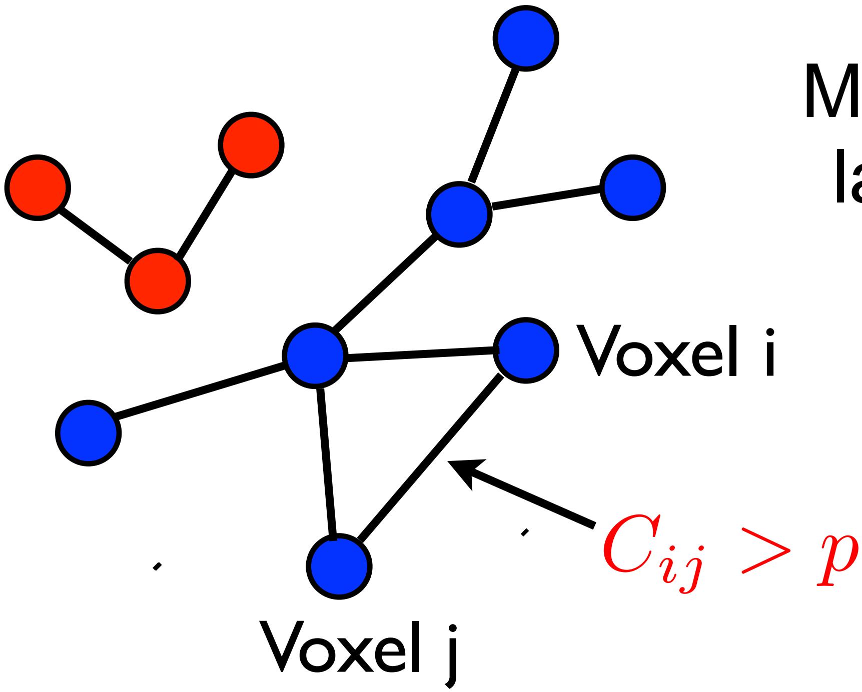
$$C_{ij} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

Connect two voxels if correlation is larger than threshold p:

$$C_{ij} > p$$

Create a functional brain network

$$C_{ij} = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

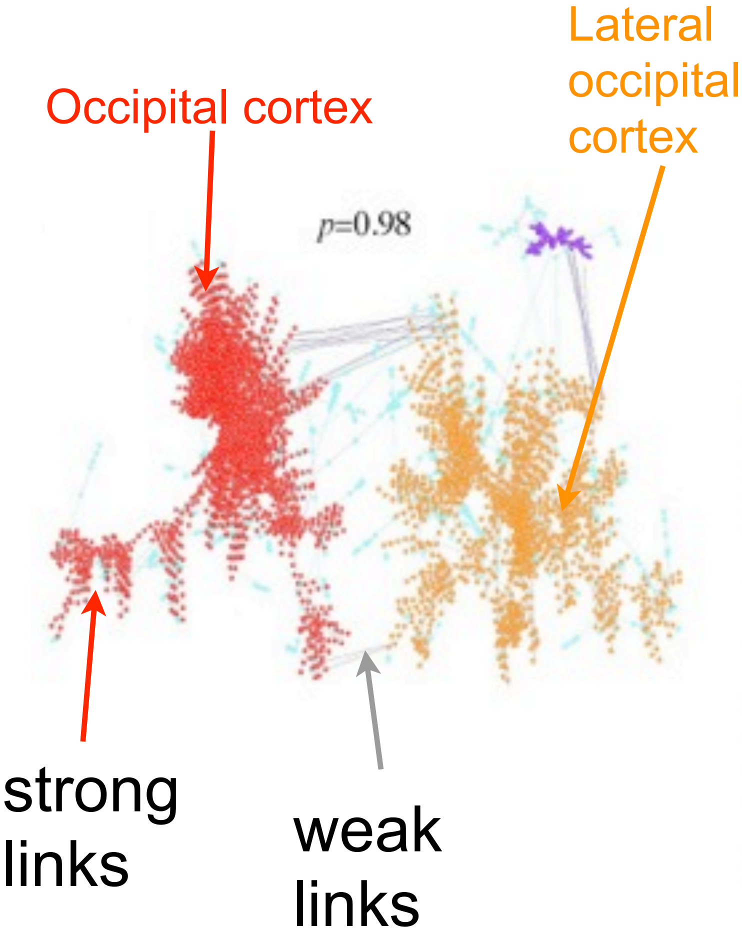


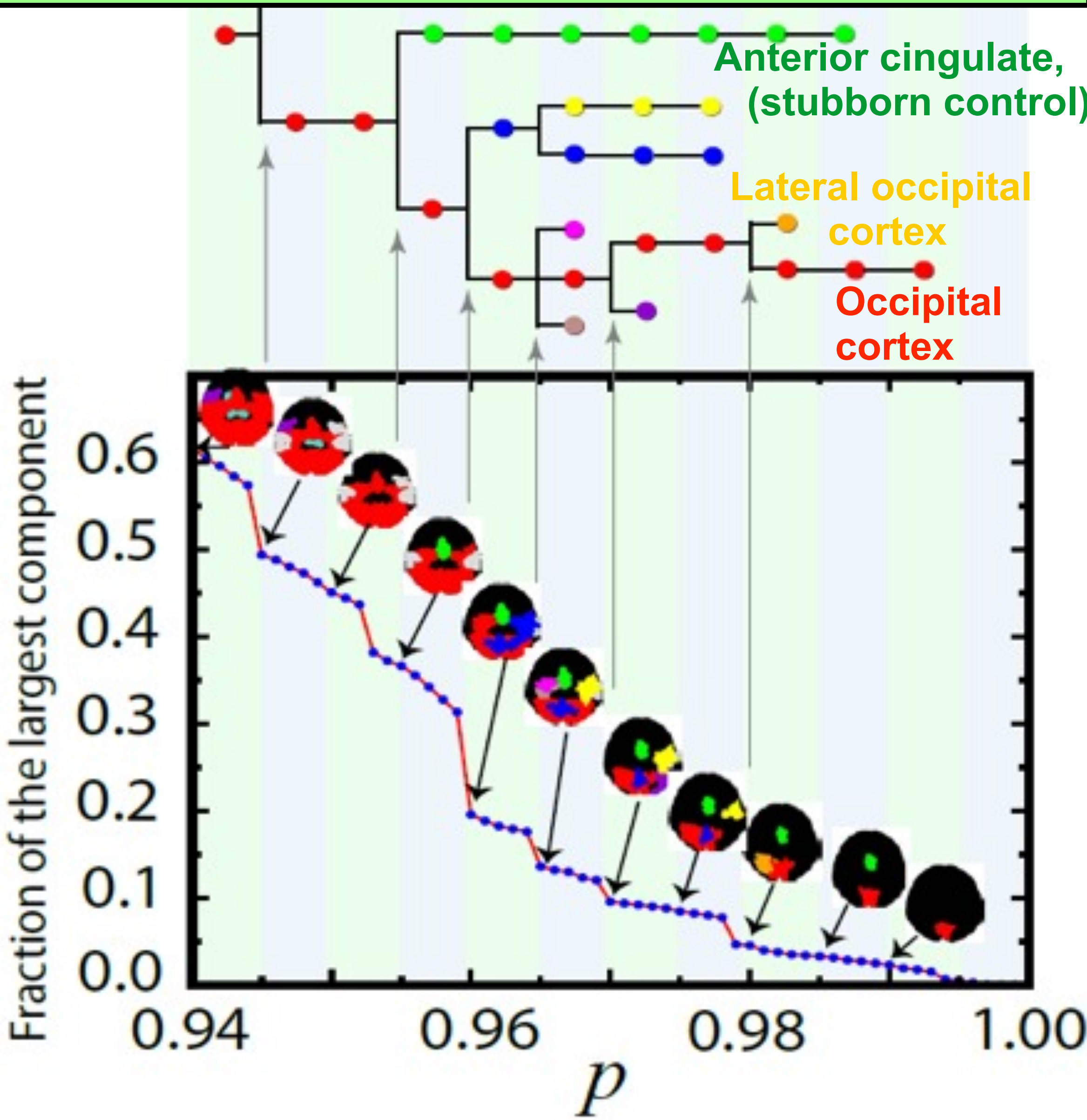
Monitor the size largest cluster versus p

How to define p?
Bond Percolation

Percolation defines a hierarchical brain NoN of strong and weak links

neither second order nor first order (Achlioptas?)

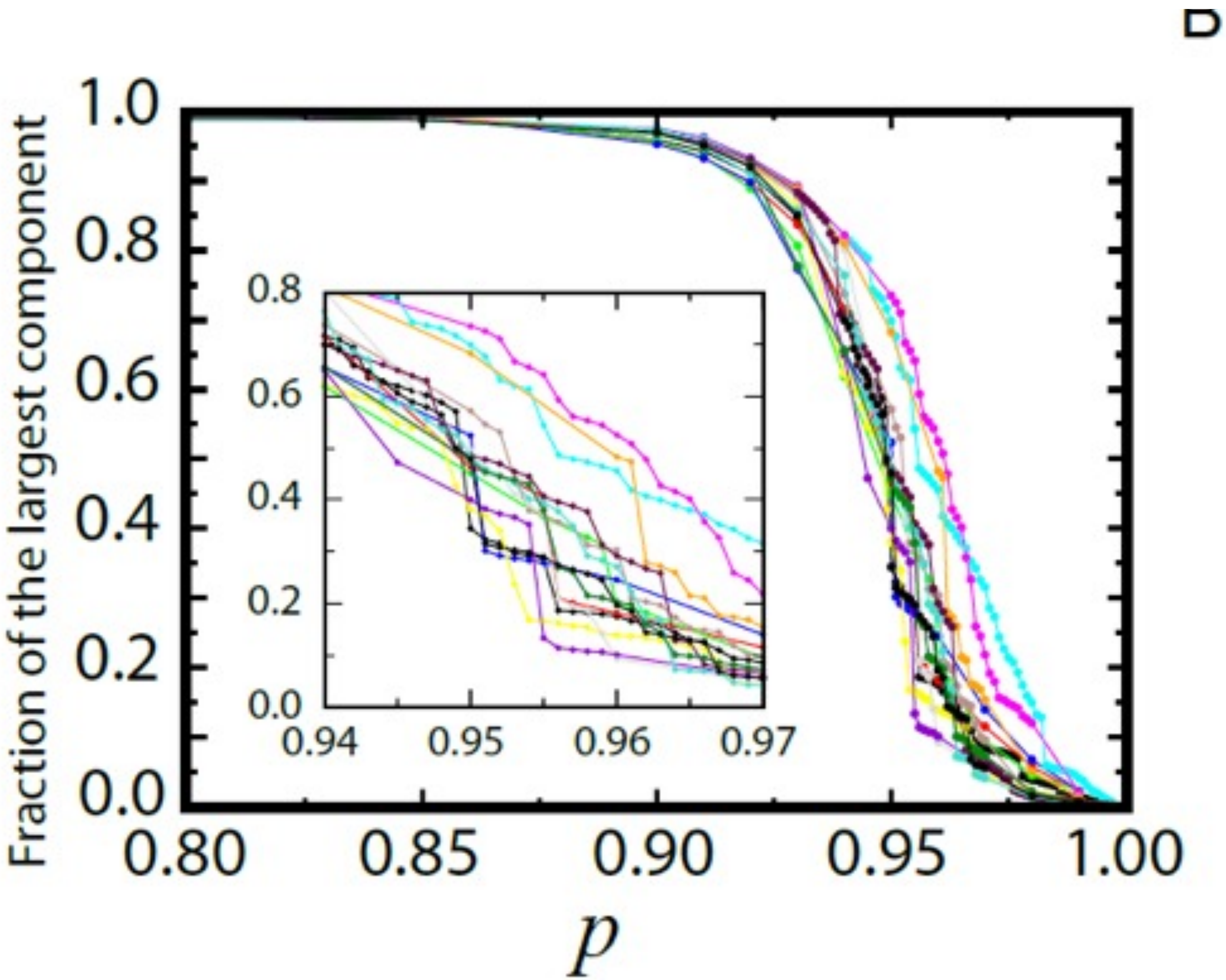




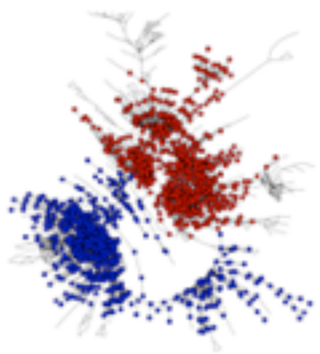
Universality:
Same for resting state in humans over all subjects and rats

Similar results over all subjects in dual task

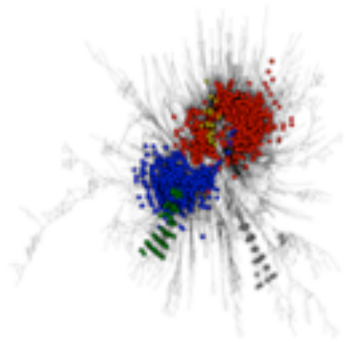
Threshold p_c is not universal



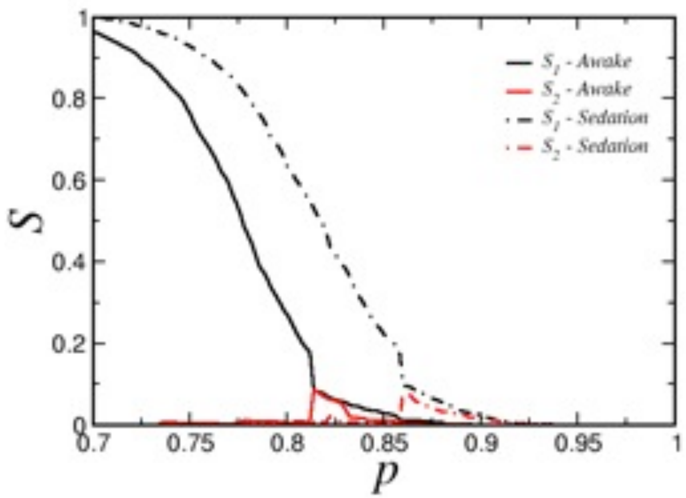
Universality: Similar results in Resting State in humans and rats



Awake



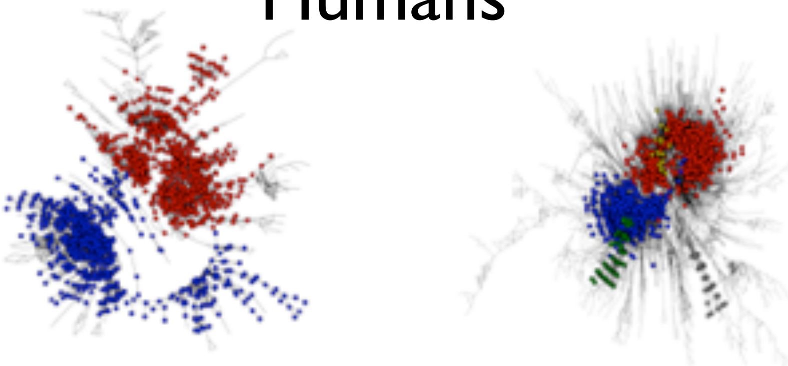
Sedation

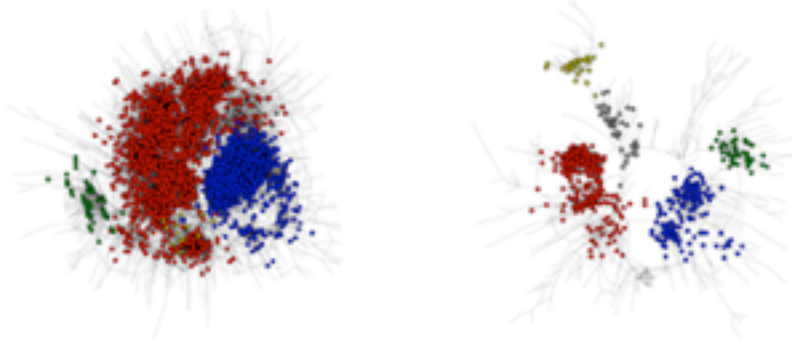


Universality: Similar results in Resting State in humans and rats

Humans

Rats



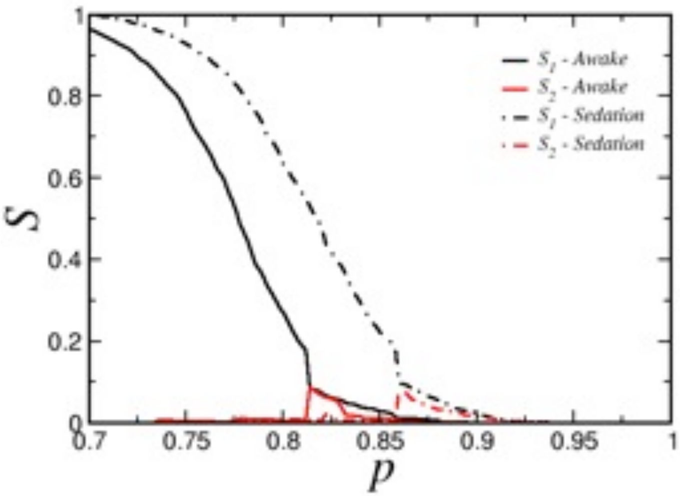


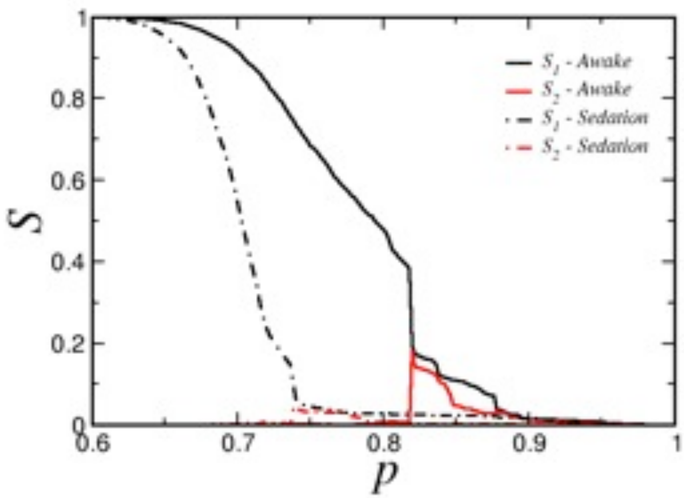
Awake

Sedation

Awake

Sedation

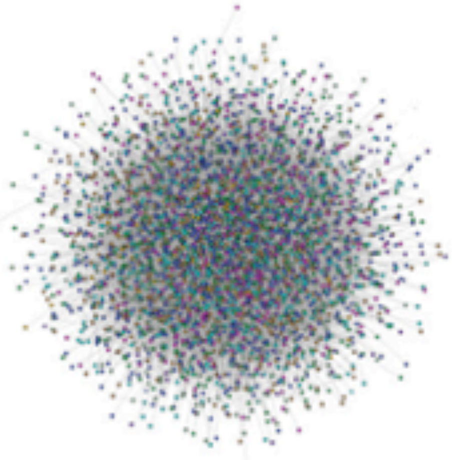




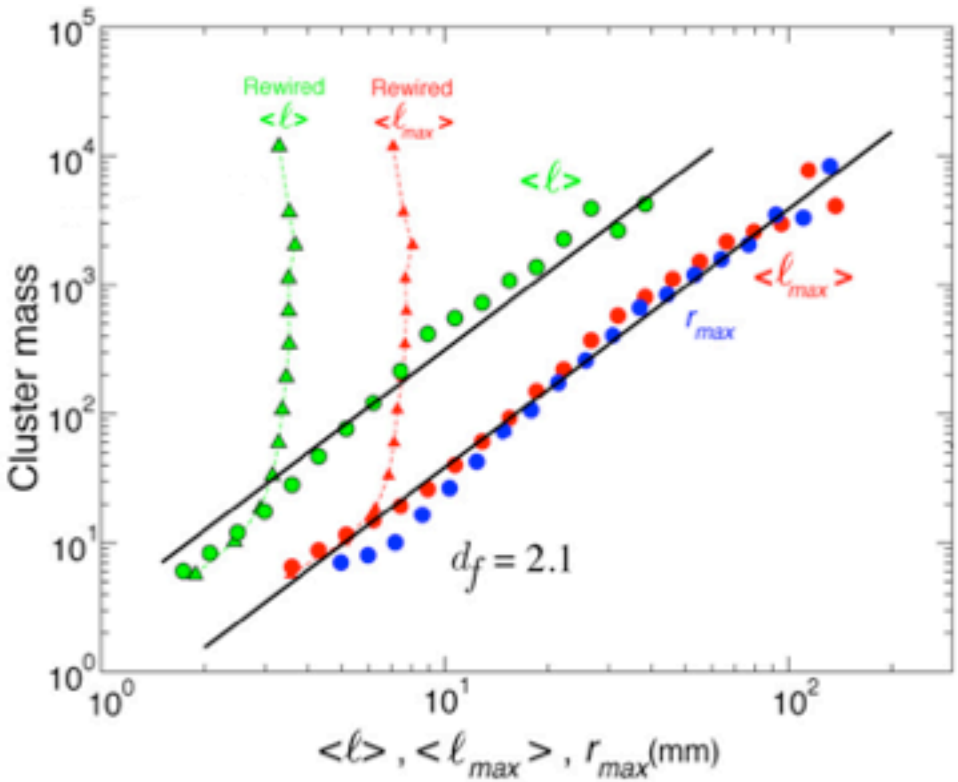
Brain networks are fractals not small-world

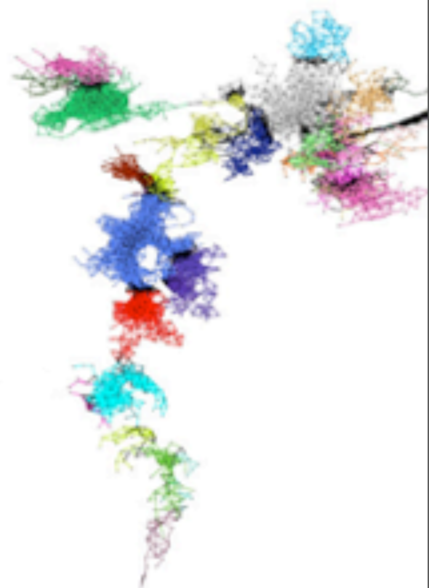
Song, Wang, Makse, Nature (2005)

$$N \sim e^{\ell}$$



Rewired
small-world network





Fractal module
in network space

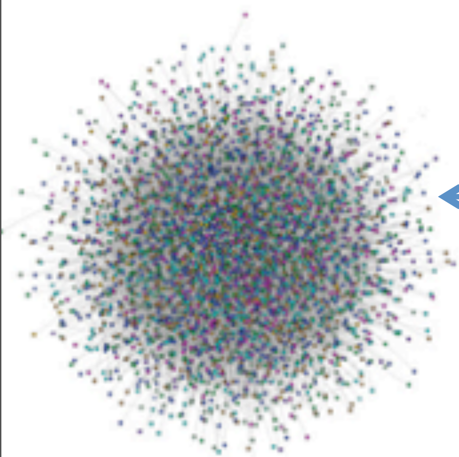
$$d_f \approx 2.1 \qquad N(\ell) \sim \ell^{d_f}$$

Brain networks are also scale-free

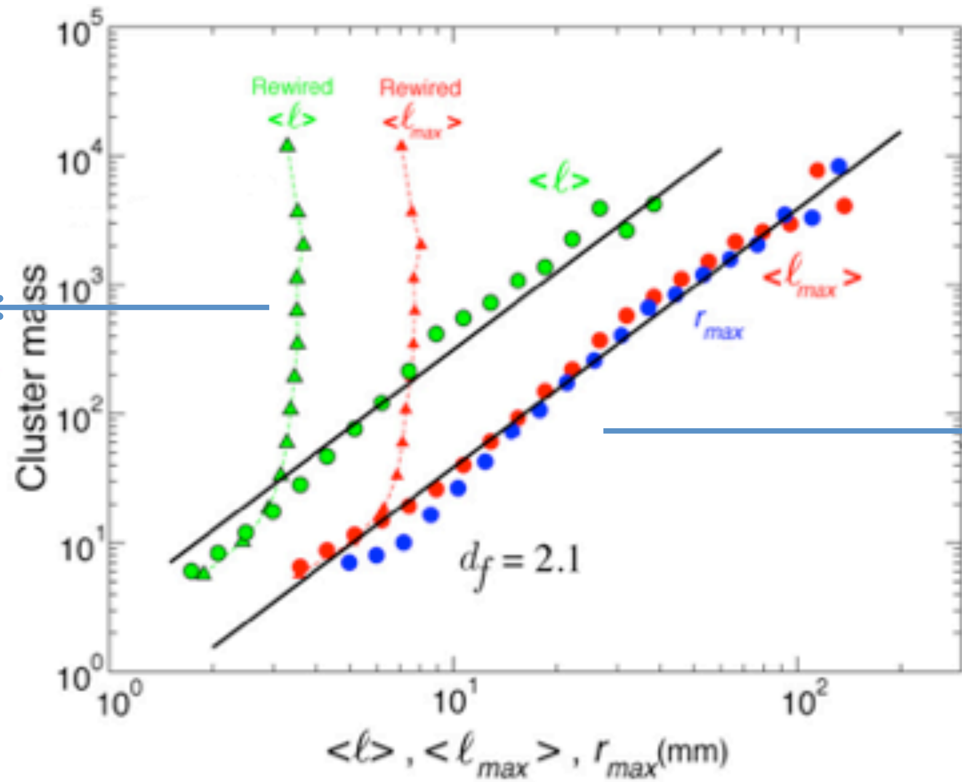
Brain networks are fractals not small-world

Song, Wang, Makse, Nature (2005)

$$N \sim e^{\ell}$$



Rewired
small-world network



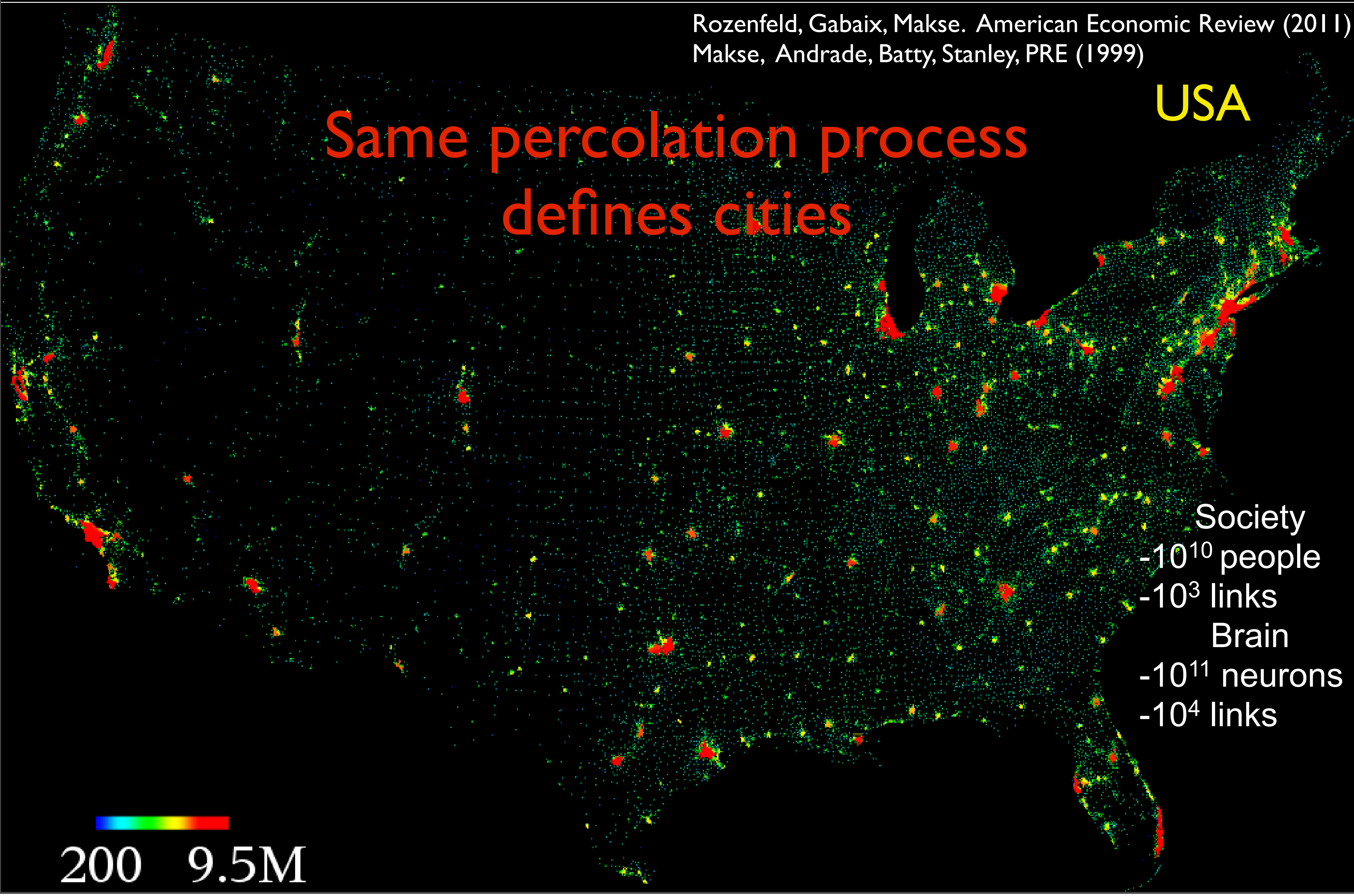
Fractal module
in network space

$$d_f \approx 2.1 \qquad N(\ell) \sim \ell^{d_f}$$

Brain networks are also scale-free

BRAIN AND THE CITY

Rozenfeld, Gabaix, Makse. American Economic Review (2011)
Makse, Andrade, Batty, Stanley, PRE (1999)



Same percolation process defines cities

Society
-10^{10} people
-10^{3} links
Brain
-10^{11} neurons
-10^{4} links

Obesity percolation

Gallos, Makse, Sci. Rep (2012)

Using CDC data at county level to investigate the spatial spreading of obesity

2004

Obese: BMI>30

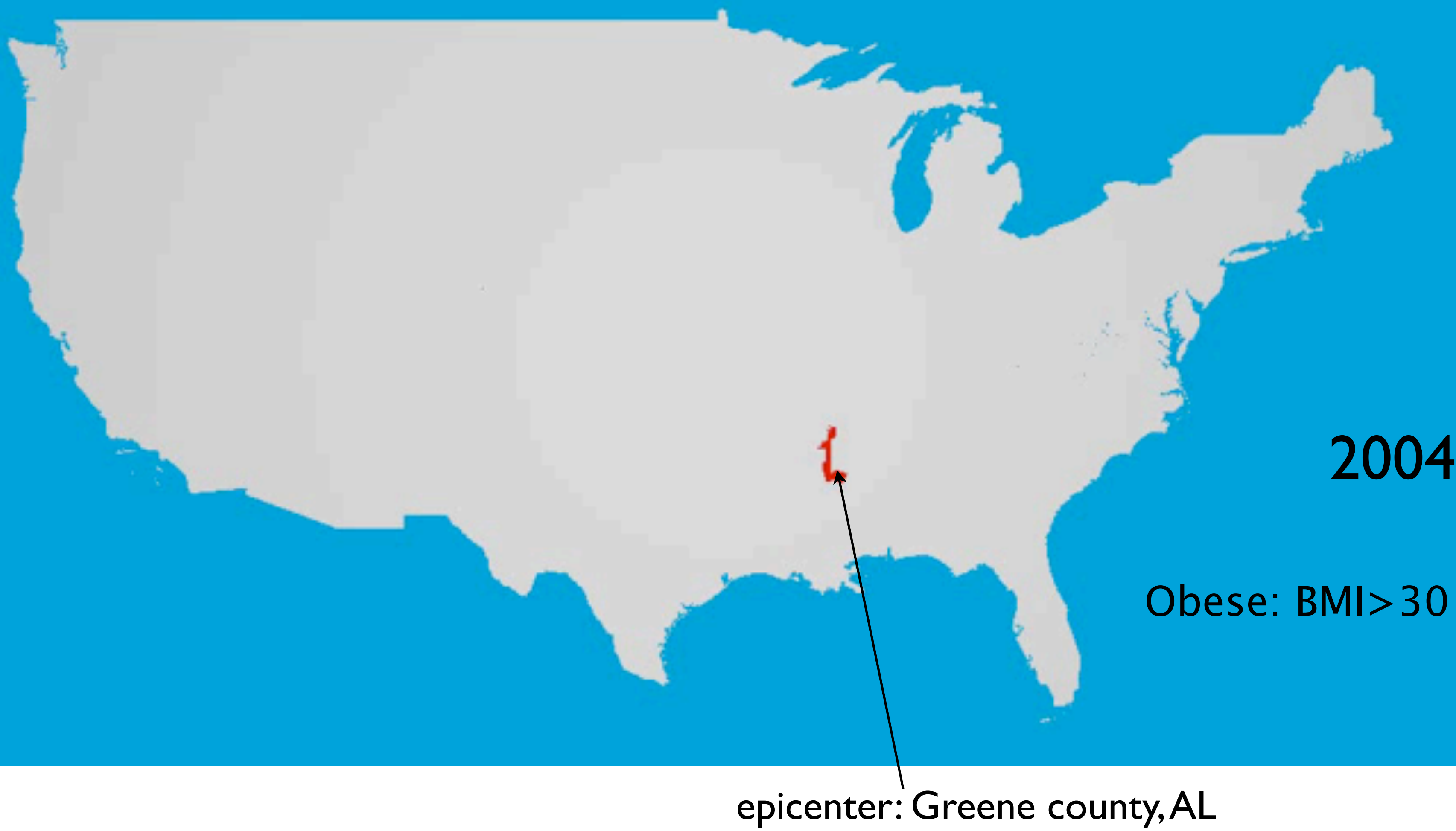
epicenter: Greene county, AL

regions of high number of obese people, BMI>30

Obesity percolation

Gallos, Makse, Sci. Rep (2012)

Using CDC data at county level to investigate the spatial spreading of obesity



regions of high number of obese people, BMI>30

Obesity percolation

Gallos, Makse, Sci. Rep (2012)

Using CDC data at county level to investigate the spatial spreading of obesity

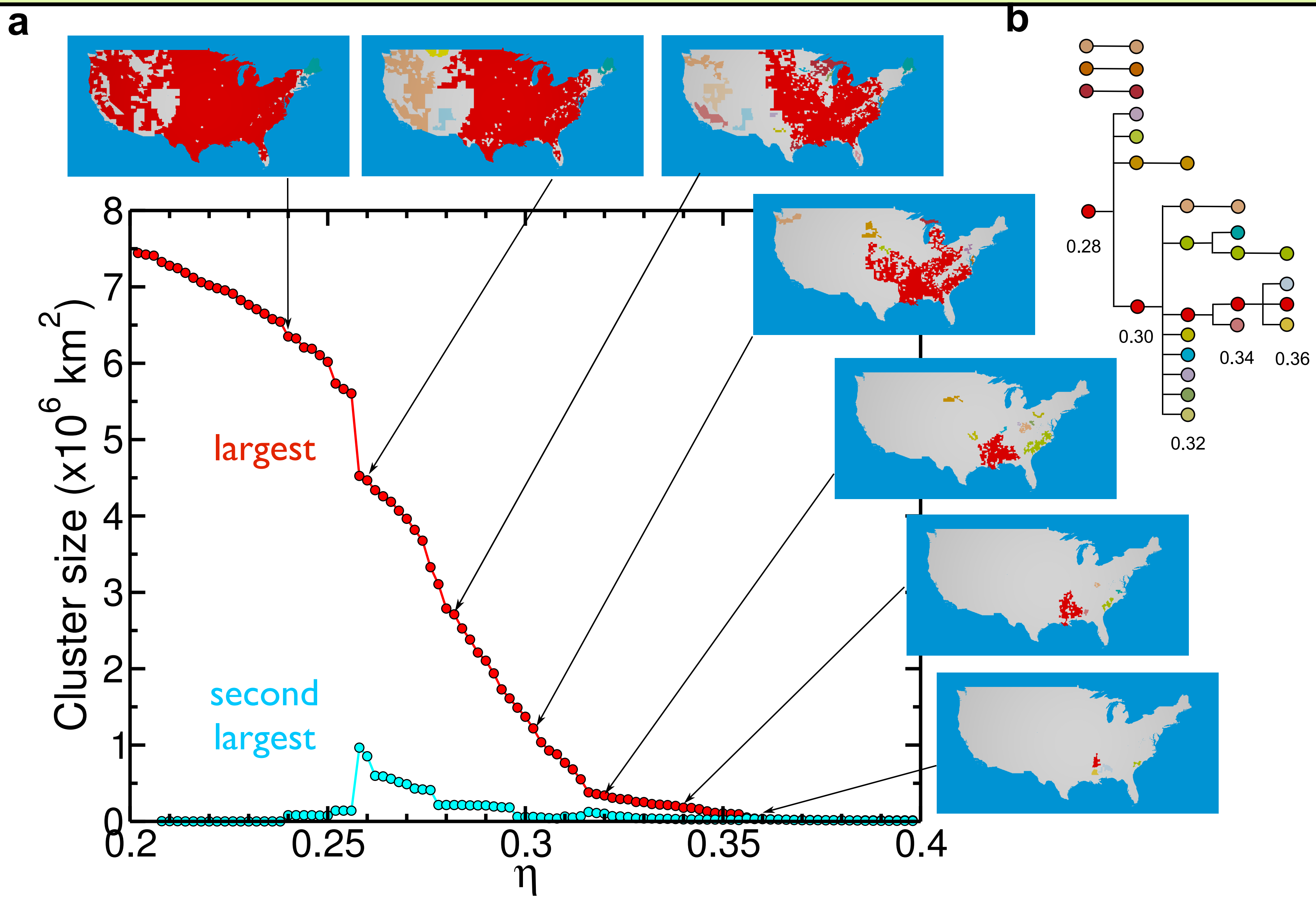
2004

Obese: BMI>30

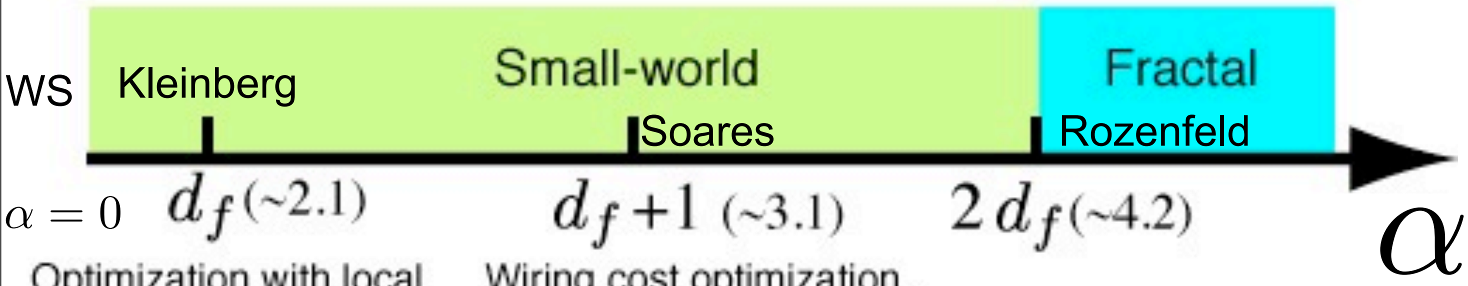
epicenter: Greene county, AL

regions of high number of obese people, BMI>30

Obesity Percolation: same process as in the brain



Navigation in a Brain NoN: what is the optimal wiring of weak links?

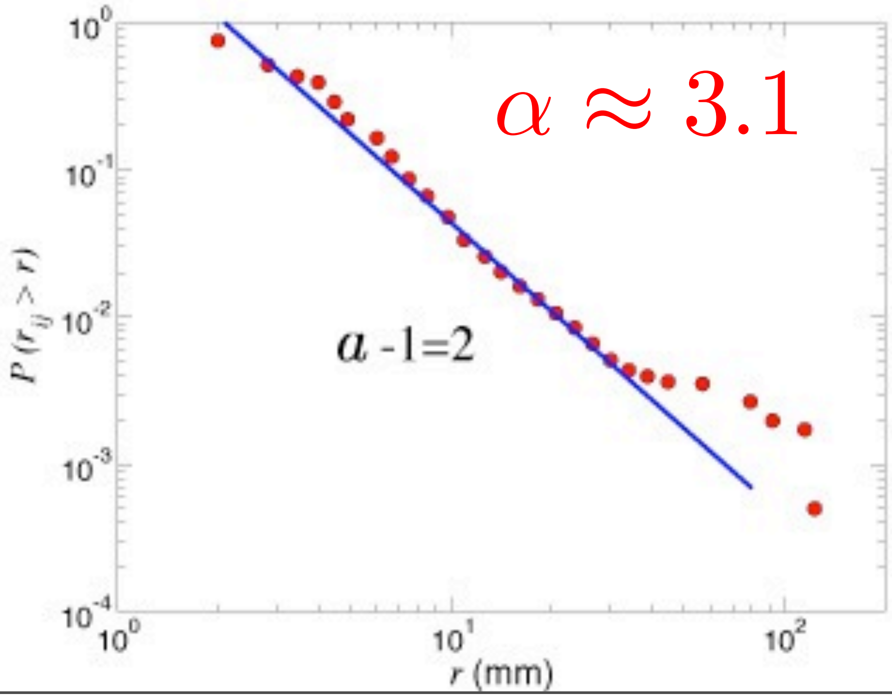


-Kleinberg, Nature (2000)
-Li, Andrade, Havlin, PRL (2010)
-Rozenfeld, Song, Makse PRL (2010)

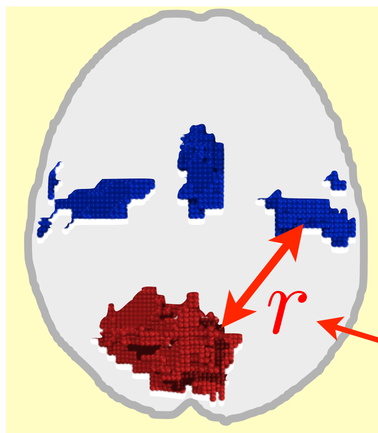
Optimization with local position knowledge (Milgram experiment, Internet) (greedy search)

Wiring cost optimization, with global routing knowledge (air-transport network)

Prof. Soares is right! Weak links are short cuts designed optimally to minimize their cost-length and maximize integration among the modules



$\alpha \approx 3.1$



$P(r) \sim r^{-\alpha}$

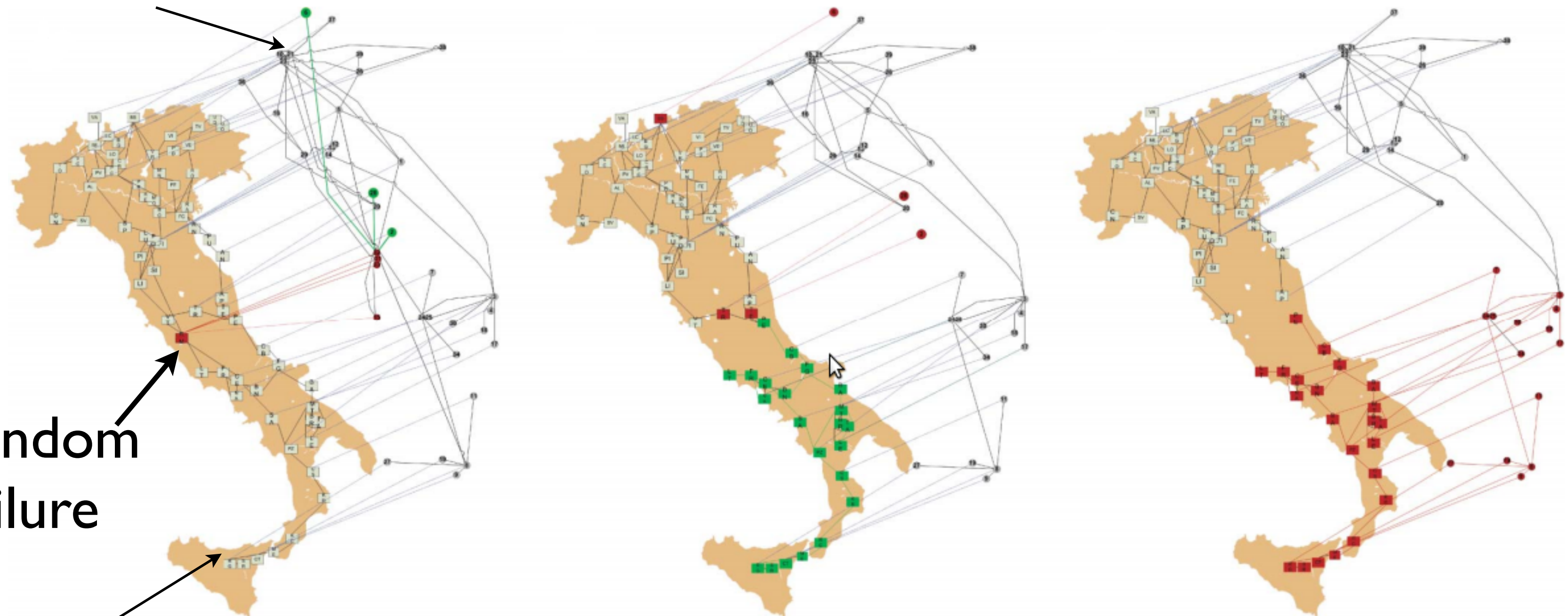
weak links/ short cuts

Next: Brain Conundrum 2
Which nodes optimally connect the Brain NoN?

Cascades of failure: two stable scale-free networks are very fragile in a NoN

Internet network

Blackout in Italy 2003



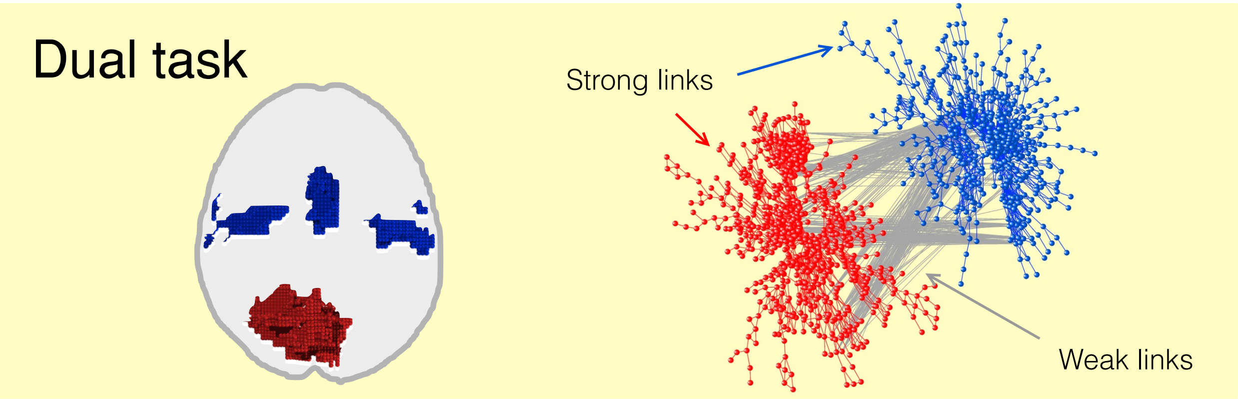
random failure

power grid network

Havlin et al. Nature (2010)
Uncorrelated NoN theory with one-to-one random interconnections

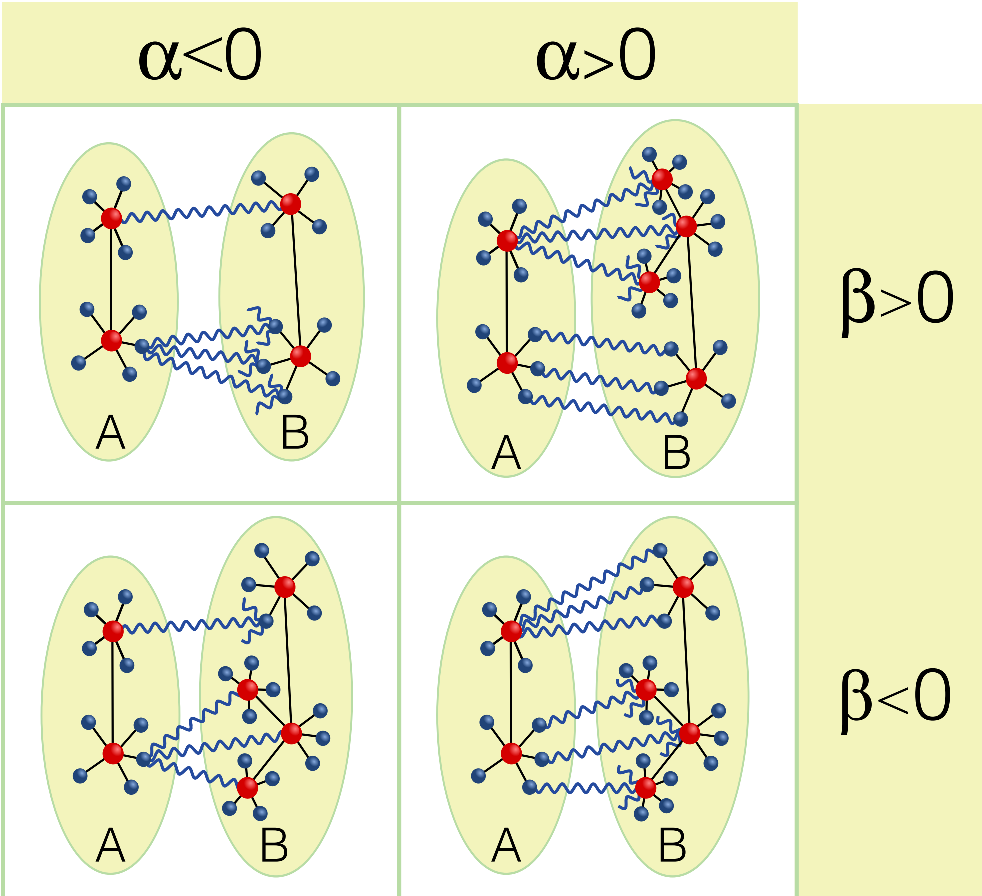
Reis, Andrade, Sigman, Canals, Makse, Nature Phys 2014

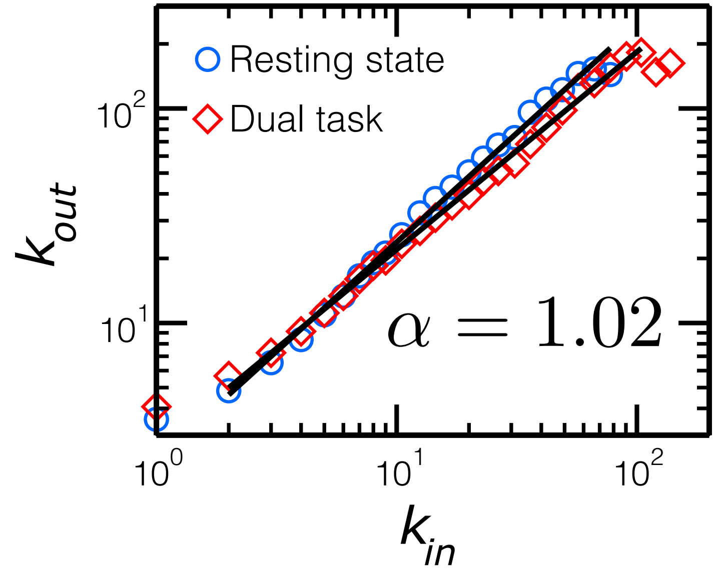
If Network of Networks are so fragile,
Why brain NoN are so stable?

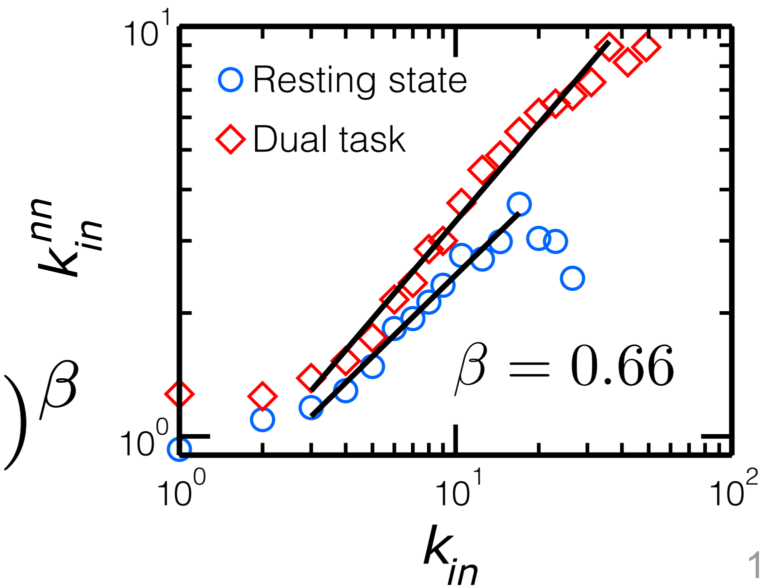


Which nodes are responsible for broadcasting information to the whole Network of Networks?
Hubs or low degree nodes?

Brain NoN have correlated redundancies

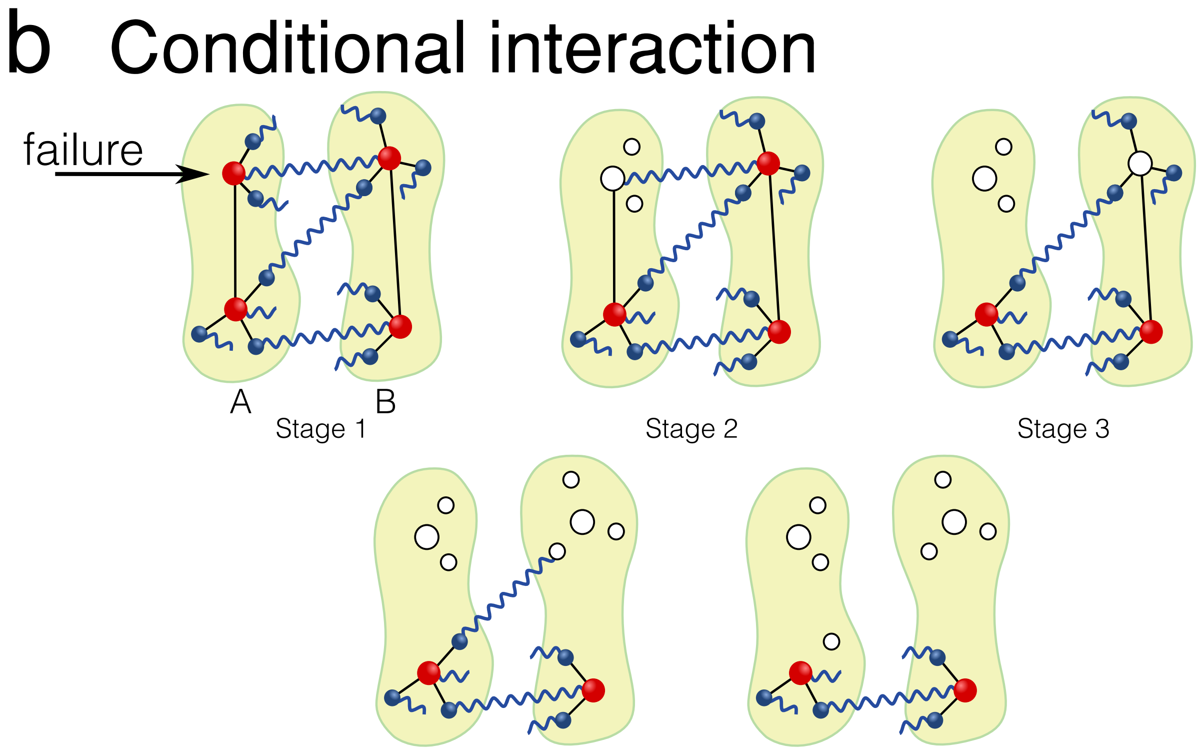






$$k_{\mathrm{out}} \sim (k_{\mathrm{in}})^{\alpha} \qquad k_{\mathrm{in}}^{\mathrm{nn}} \sim (k_{\mathrm{in}})^{\beta}$$

Correlated percolation theory of random failure to test stability under failure

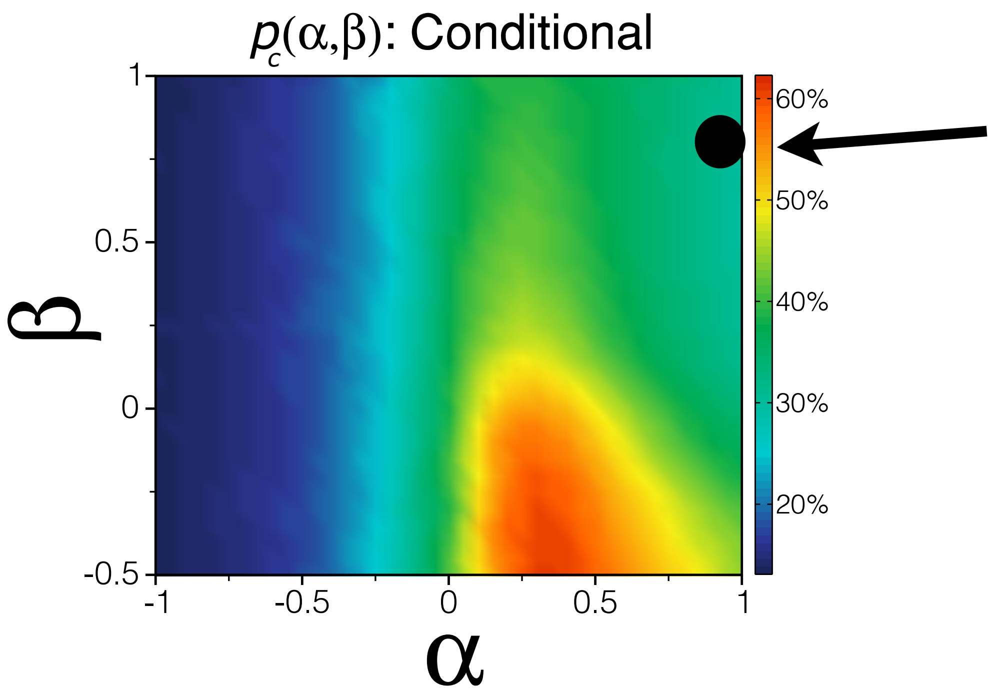


Calculate p_c under cascading failure of nodes chosen at random.

Low p_c is optimal:
more robust structure and faster information transfer

Brain NoN are super-optimal
Superspreaders in NoN are the hubs

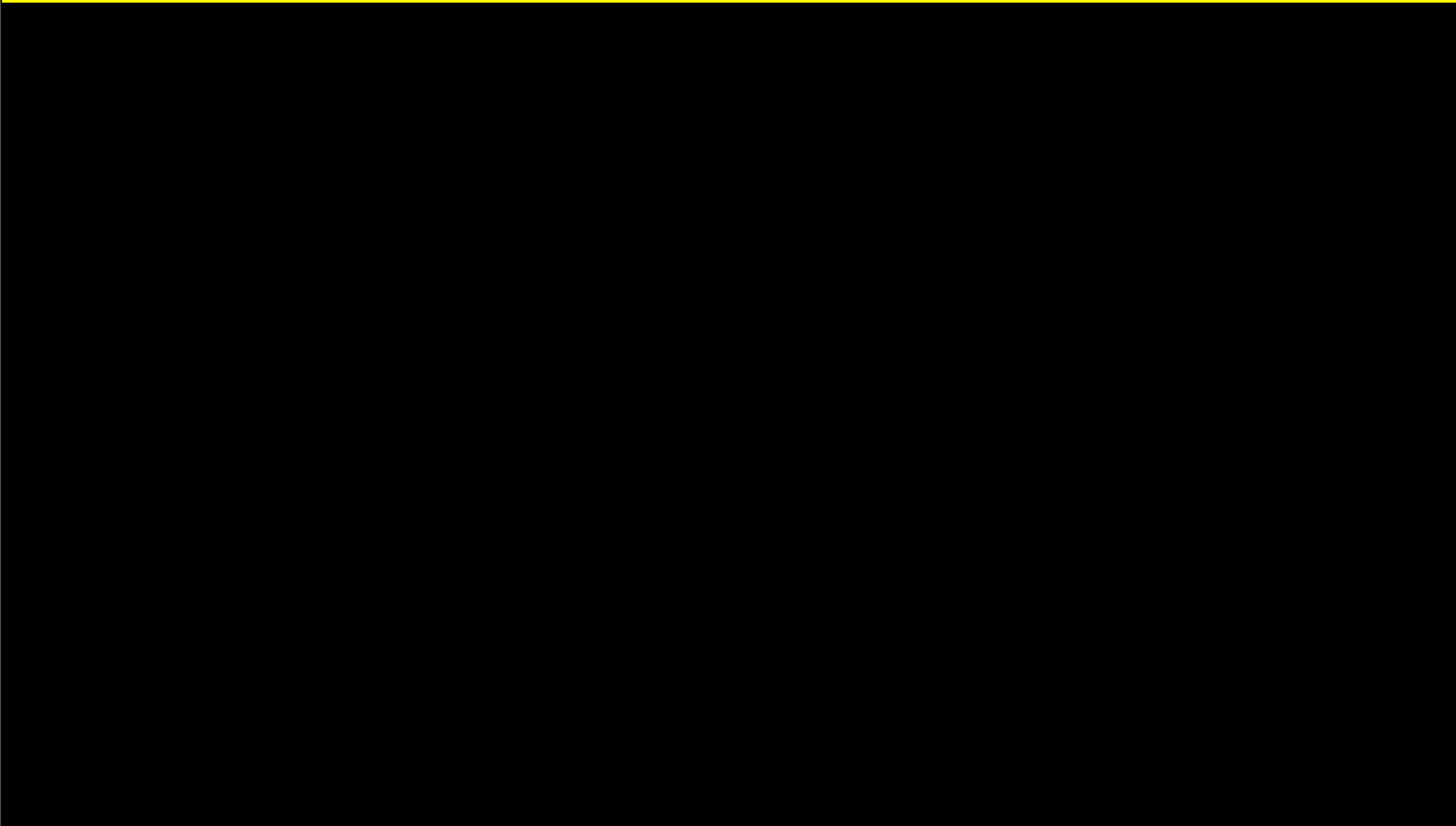
Correlated Brain NoN is Optimal for stability:
the less vulnerable structure corresponds to hub-hub connections between networks



$$\alpha = 1.02$$
$$\beta = 0.66$$

Optimal for stability and information transfer

3. Emergent collective behavior from eye-tracking



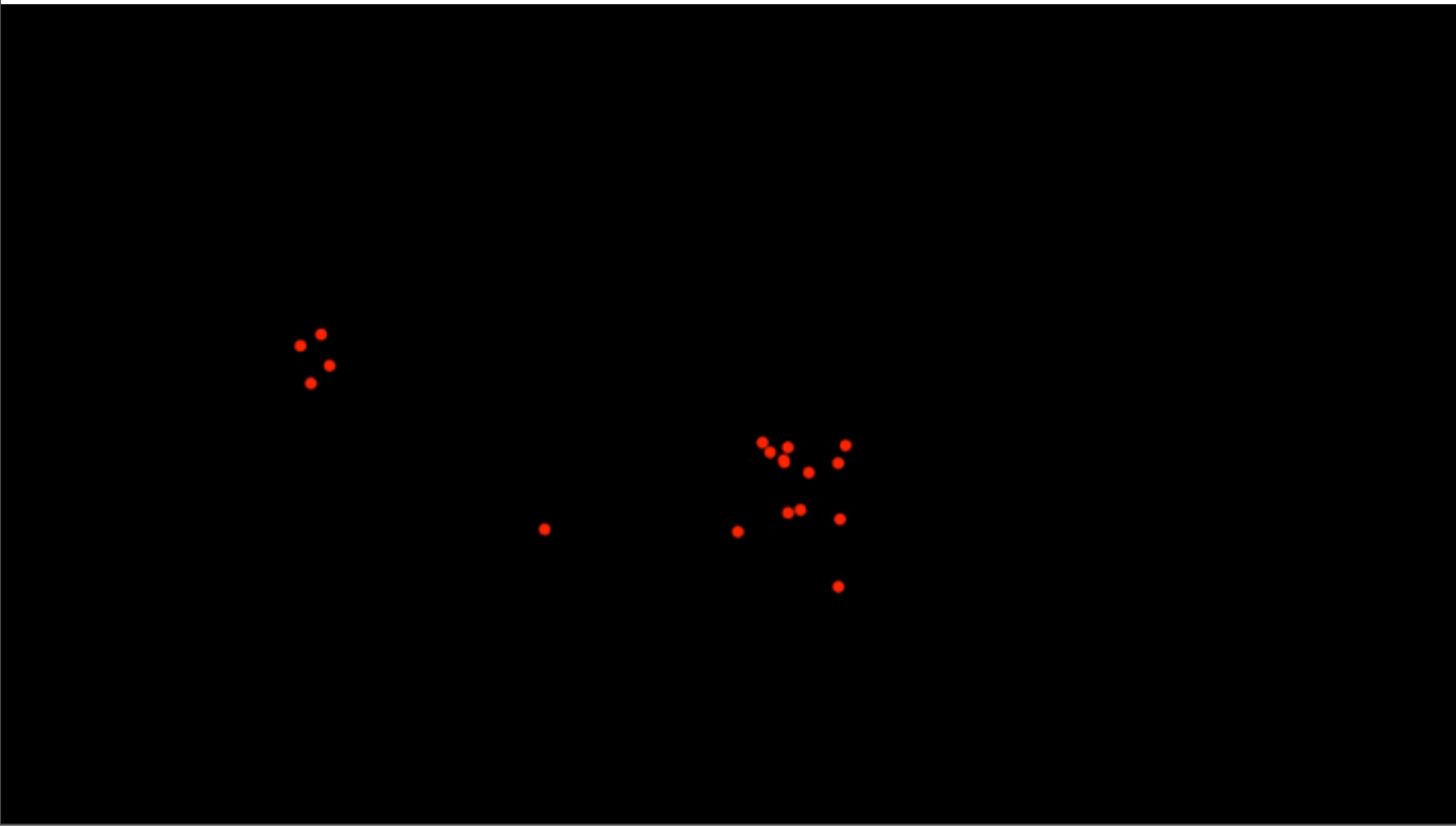
Inspired by collective behavior in starling flocks

Cavagna et al, PNAS 2010

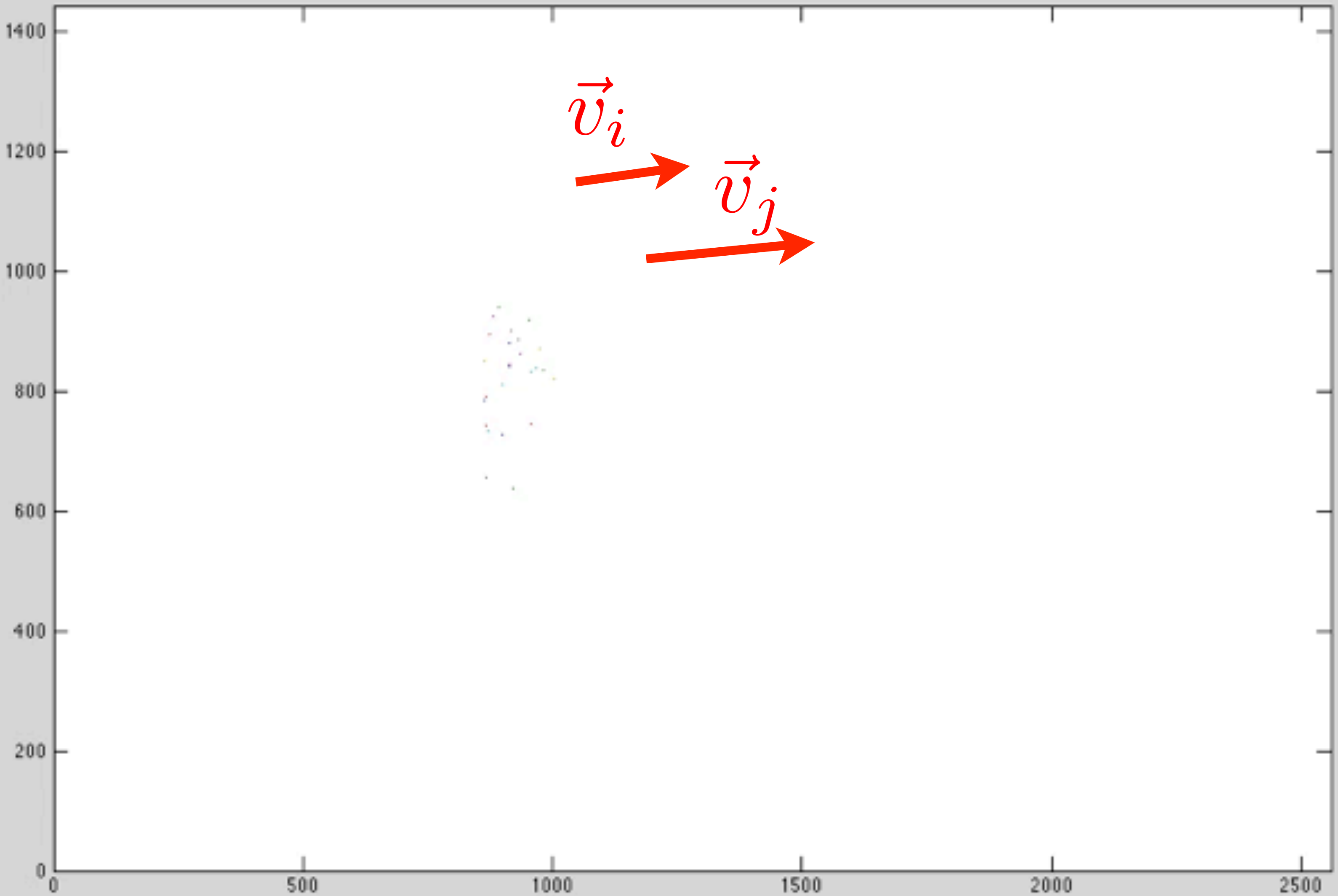
Understanding “engagement” of a video

Eye-tracking (measure the eye movement) for 25 viewers of SuperBowl 2014 ads

Lucas Parra, CCNY

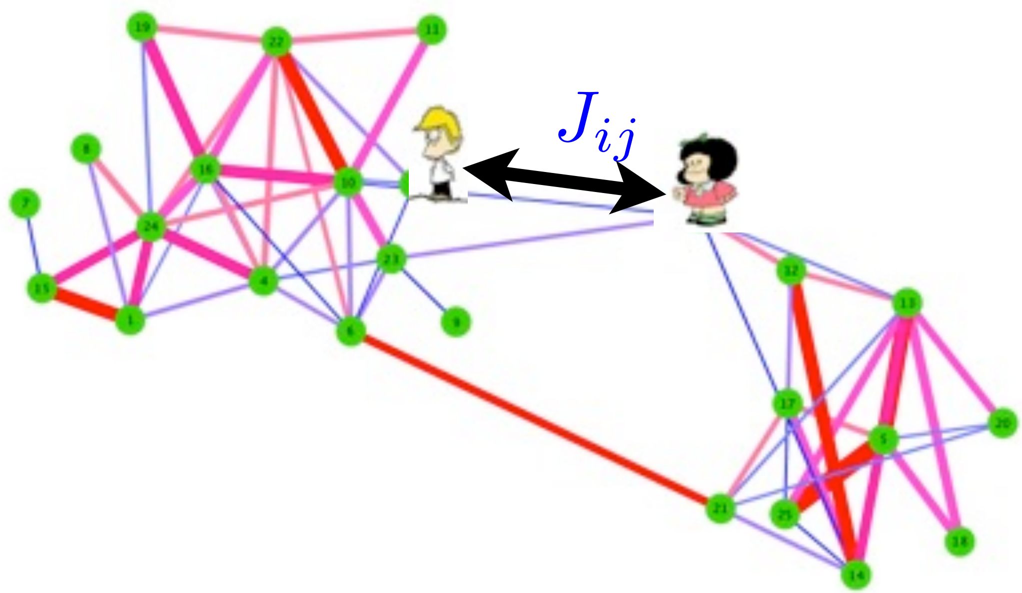


Eye-movement trajectories



Mapping to a fully connected XY spin-glass to infer pair-wise "interactions" = "homophily"

network of "homophily" through the video



Hamiltonian

$$\mathcal{H} = -J_{ij}\ \vec{v}_i \cdot \vec{v}_j$$

pair-wise interactions

Inferring J_{ij} from the correlation function:

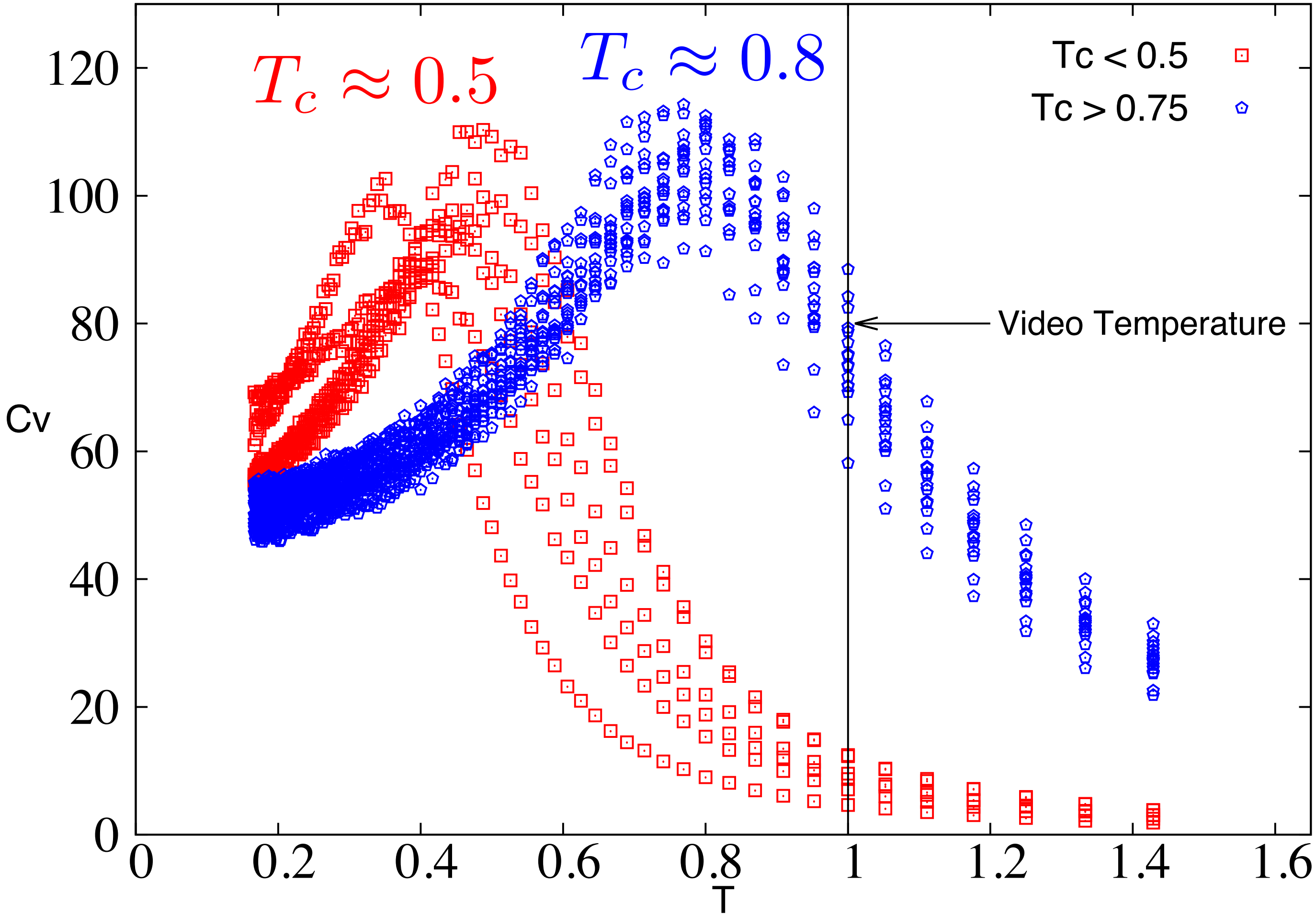
$$C_{ij} = \langle \vec{v}_i \cdot \vec{v}_j \rangle - \langle \vec{v}_i \rangle \cdot \langle \vec{v}_j \rangle$$

Maximum entropy methods: Bialek, 2010

Then: Calculate the partition function

“Specific heat” C_V revels two groups of videos

Measuring the “alertness” or “engagement” of a video as the closeness to the critical temperature:



Critical videos present larger homophily and have larger TV ratings

Summary: an architectural law for functional brain networks

1. The functional brain organizes into a NoN made of strong and weak links.

2. The spatial arrangement of weak links is optimal for information transfer minimizing wiring cost.

3. Network hubs are responsible for broadcasting information to the whole network.

4. The resulting correlated NoN is optimal for vulnerability under random failure in contrast to uncorrelated NoN with one-to-one connectivity.