Collective correlations of Brodmann areas from random matrix theory denoising of fMRI data

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Outline

- Mark Kac Complex Systems Research Center and why have we decided to study brain?
- Random Matrix Theory as a probability calculus for the XXI century
- Wishart ensemble
- Spectral analysis of the sample fMRI data [Zdzisław Burda, Jennifer Kornelsen*,Maciej A. Nowak, Bartosz Porębski, Uta Sboto-Frankenstein*, Bogusław Tomanek*, Jacek Tyburczyk; arXiv:1306.3825, Acta Phys. Pol. B44 (2013) 1243.]
- Prospects and future plans

Mark Kac Complex Systems Research Center

- Mark Kac CSRC - Scientific centre for advanced research, creating the methodology for the developing branches of science connected to computer technologies and at the same time, promoting this methodology via practical (industrial) applications in high-tech sector.
- Main areas:
  1. Computer and telecommunication networks (NetLab)
  2. Financial, market and crises risk management and risk assessment (RiskLab)
  3. Bioinformatics and genetic computer studies (BioLab)
  4. Quantum information and fundamental studies of strongly correlated atoms (QuantLab)
  5. Cognitive science, brain signal analysis, computer games. (CogniLab)

RMT -Ubiquitous applications in practically all branches of science.

- Microscopic universality (e.g. Wigner surmise) versus macrocopic bulk (e.g. Wigner semicircle), nonlinearity
- **Random matrix theory is a sort of probability calculus, where the random variable is a whole matrix**
- Classical pdf's are replaced in RMT by the spectral pdfs (eigenvalues and eigenvectors)
- In the limit when dimensions of the matrix tends to infinity, theory simplifies and analogy to classical probability theory becomes exact in the mathematical sense (free random variables theory [Voiculescu, 1986])
- In practice, $\infty = 8$

Eigeninference and information retrieval based on RMT

- Quantitative finances [Bouchaud et al; Stanley et al., 1999] - see e.g. recent review [Burda et al, Quantitative Finances 11, Issue 7:1103, 2011]
- Wireless telecommunication [Foschini,Telatar; 1996] - see e.g. recent book [Couillet and Debbah, Random Matrix Methods for Wireless Communications, Cambridge University Press, Cambridge, 2011].
- Genetics [PV. Dahirel et al., Coordinate linkage of HIV evolution reveals regiosn of immunological vulnerability PNAS 108: 11530-11535, 2012.]

"Where is the **brain**?"

- Nomhermitian RM for MEG data [Ioannides et al, 2000]
- Universal RMT pattern in EEG [Seba; 2003]
- Synchronization patterns for seizures in epileptic attacks [Osorio, Y-C Lai, 2011]
- ???
- Implicit "matrix models" - PCA, spectral analysis of adjacency matrices, non-backtracing operators for sparse systems
- Lack of massive, systematic approach based on RMT to the neuroinformatics

Wishart Ensemble, 1928

- Multivariate statistics of generic measurements X_{it}, where "position" $i = 1, \dots N$, "time" $t = 1, \dots T$
- Correlation matrix $C_{ij} = \frac{1}{T} \sum_{i=1}^{T} X_{it} X_{jt}^{*} \equiv \frac{1}{T} [XX^{\dagger}]_{ij}$
- If each X_{it} is real (complex) Gaussian, we have real (complex) Wishart Ensemble (LOE/LUE) - exactly solvable system.
- Dramatic simplification if N and T tend to infinity, while $r = N/T$ is kept fixed (we choose $r \leq 1$)
- Link to FRV theory (free Poisson process)
- Spectral density $\rho(\lambda)$ given by Marchenko-Pastur fomula $\rho(\lambda) = \frac{1}{2\pi r} \frac{\sqrt{(\lambda_{+} - \lambda)(\lambda - \lambda_{-})}}{\lambda}$ where $c_{\mp} = (1 \mp \sqrt{r})^2$.

MP distribution

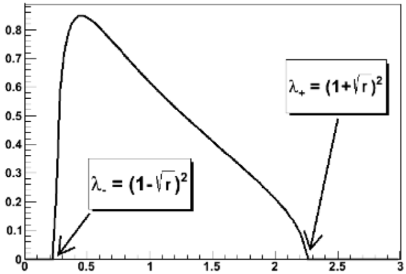


Fig. 1: Marchenko-Pastur distribution for $r = 0.256$.

Lesson on inference from MP formula

- Even in the case when "true" covariance matrix is a unit matrix, the spectrum is always "broad", i.e spans the interval $[(1-\sqrt{r})^2, (1+\sqrt{r})^2]$.
- Every finite time multivariate measurement introduces the noise
- Exact result retrievable only in the academic, idealized limit N fixed, $T = \infty$, i.e. $r = 0$, then $\rho_{approx}(\lambda) = \rho_{true}(\lambda) = \delta(\lambda - 1)$.
- Even worse, when $N = T$ $(r = 1)$ we pick the (integrable) singularity at $r = 0$.
- For $N > T$ (e.g. fMRI) the inference is contaminated by $N - T$ zero modes
- In practice, we have often only ONE matrix instead of the statistical ensemble of matrices (e.g. a hurricane, particular realization of the stock market, snapshot of the brain of an individual) – we believe into "ergodicity" (self-averaging of the huge matrix)

Simple exercise, due to help of collaborators from NRC of Canada Institute for Biodiagnostics

- Subject: 61 year old healthy male, right-handed
- MR data acquisition: 3 Tesla Siemens Trio, BOLD techniques used etc...
- Motor task: REST vs TAP FINGERS
- 163 volumes acquired (tapping 1-22, 43-62, 83-102, 123-142)

Data structure

- "Brodmannization" of the data: Reduction of N based on cytoarchitectonics (in practice, we used 41 out of 52 regions due to small sizes). This procedure yielded the $r \sim 0.256$, i.e. amenable to RMT analysis.
- Visualization of the data we based on the Talairach atlas, i.e. translation/rotation and non-linear warp (shrunk) of the scan into the standartised dimensions of the brain ($L = 172$mm, $H = 116$mm, $W = 136$mm))
- Consistency test: We standartise the data and we reshuffled them, killing all the correlations, and we recovered the MP distribution (null hyphotesis: zero information - pure (Gaussian) noise)
- Deviations from MP distribution are interpreted as a benchmark of true correlations.

Eigenseries corresponding to the largest eigenvalue

- Clear "outlier", $\lambda_1 = 22,6$ comparing to $\lambda_+ = 2.25$.
- We calculated participation ratio of the eigenvector $PR(v) = \frac{1}{\sum_{i=1}^{N} v_i^4}$=34.393
- **Recall**: All but one element of eigenvector are zero, then $PR = 1$ (localization).
  Other extreme, all elements have the same value, $PR = N$
- Clearly, λ_1 corresponds to delocalized state: all Brodmann areas are active

Idle vs Motor tasks for the largest eigenvalue

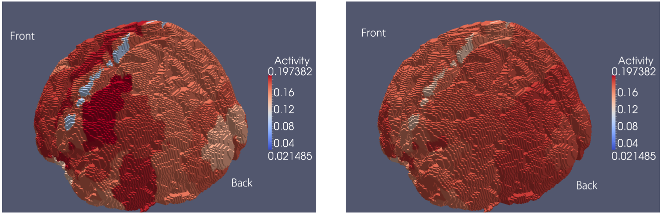


Figure: No dramatic difference between the idle and motor task functions for the first eigenvalue.

Eigenseries corresponding to the second largest eigenvalue

- Still "outlier", $\lambda_1 = 4.478$ comparing to $\lambda_+ = 2.25$.
- We calculated participation ratio of the eigenvector $PR(v) = \frac{1}{\sum_{i=1}^{N} v_i^4}$=16,42
- **Recall**: All but one element of eigenvector are zero, then $PR = 1$.
  Other extreme, all elements have the same value, $PR = N$
- Less than a half of Brodmann areas take part in the corresponding brain activity.

Idle vs Motor tasks for the second to largest eigenvalue

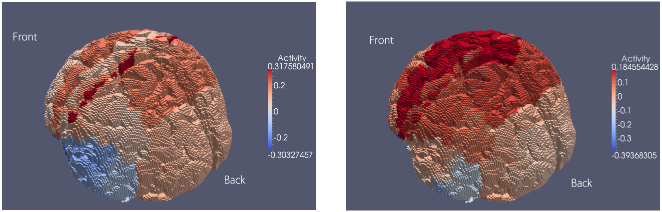


Figure: Clear difference between the idle and motor task functions for the second eigenvalue.

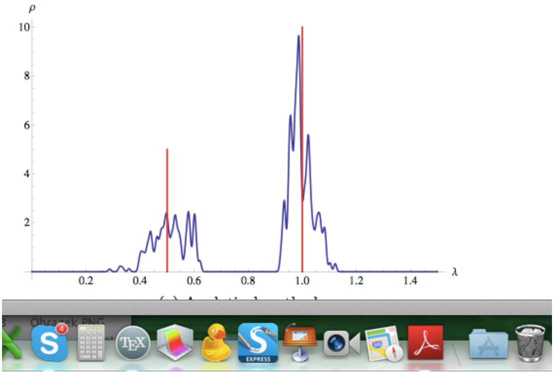
Tapping versus idle - continued

- Largest deviations can be seen between between Brodmann areas 9,10,25 (prefrontal cortex), or more precisely, *Dorsolateral Prefrontal C.*, *Anterior Prefrontal C.*, *Subgenual C.* and 47 (frontal lobe, *Inferior Prefrontal C.*)
- Consistent with the belief, that prefrontal cortex is responsible for motor task planning.

Can we do better?

- Correlated Wishart ensemble $P(X)dX \sim e^{-TrXB^{-1}X^{\dagger}A^{-1}}dX$ where $< X_{it}X_{jt'} >= A_{ij}B_{tt'}$.
- Example: A. Gabrielli's "block Gaussians" for mice Broadmann areas represent the case $B = \mathbf{1}_T$, $A = \mathrm{diag}(\sigma_1, ...\sigma_K)$ with $p_i = n_i/N$.
- FRV calculus and Padé approximants can provide eigeninference, see, e.g. [Drogosz, Jurkiewicz, Łukaszewski, Nowak, Phys. Rev. E 92, 022111 (2015)]

Example



Generalizations

- Non-Gaussian entries (Lévy, Student-Fischer, polynomial...)
- Finite size effects
- Statistics of large deviations
- Δ-lagged and cross-correlations between matrices X (e.g. structure) and Y (e.g. function) - complex spectrum
  $C_{ij}(\Delta) = \frac{1}{T} \sum_{i=1}^{T} X_{i,t} X_{j,t+\Delta}$,
  $R = XY^\dagger$
- Relation to ICA?
- Matrix-valued stochastic processes (highly nonlinear spectral phenomena)

Prospects

- 1364+650=2014
  $\longrightarrow$ New Campus of the JU; Biotechnology, Mathematics, Physics, Psychology and Neuroergonomics within walking distance,
- Mark Kac Center started collaboration with Adolf Beck Labs (dAEEG, fMRI, EOG, unfortunately, no MEG)
- Update of Mark Kac supercomputer SHIVA, arrival of PROMETHEUS
- We started very enjoyable collaboration with Dante Chialvo and we are open for broad international collaboration in the area of brain studies
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